Application for Special Spring Session

Year V — OTIS 2019-2020

Evan Chen

March – May 2020

§1 About the Special Spring Session

Because of the widespread school closures due to COVID-19, I decided to open a number of additional spots for OTIS Correspondence from **March 23, 2020 to May 25, 2020**. This is only suggested for students who expect to have a significant amount of free time, as usually eight weeks (concurrent with high school) is not enough time to make much progress towards solving olympiad-level problems.

The details of the application are as follows:

- For most students, only the **Correspondence** format is available. You will have access to the materials and office hours, but not to the graded practice exams[1] (A very small number of Biweekly spots are possible with other instructors, but there are really not many of them, so don't count on it.)
- The cost is $240, the same as for students who joined for the spring semester. (So again, this is only really worth it if you expect to have a lot of time.)
- There is no deadline; decisions are conducted on a rolling basis. You can expect a decision within 48 hours of submitting a complete application, and you will have until May 25, 2020 to work through as much as you can.

For details of the program, please check the syllabus:

`evanchen.cc/upload/otis-syllabus.pdf`.

§2 How to apply

To apply, you should complete the problems listed below. Then, send your solutions as a PDF attachment to `evan@evanchen.cc`.

Your email must also provide the following demographic information:

- Name and grade level
- Country (and state if you are from USA)
- Past contest results and history (e.g. past USA(J)MO scores)
- Statement of purpose: say a couple sentences about about yourself, why you want to do OTIS, and what you're aiming for this year.
- A single PDF attachment with your solutions, at most 10 megabytse.

You can expect a decision within 72 hours of submitting a complete application.

[1] Although you can download the practice exams for the year and try them on your own time.

§3 Instructions on solving

If you attended OTIS or MOP in any previous year, you do not have to do the problems; just follow the instructions above, omitting the PDF attachment.

1. Try to solve as many problems as you can.
2. **You must write the solutions yourself**; don't copy-paste someone else's work.: If you have seen a problem before, you may write any solution you remember.
3. **You can ask me for help if you're stuck on something!** Just send me an email telling me what you've tried, and I'll try to push you in the right direction. This is how OTIS works for admitted students, so why not practice now?

   For the geometry problems from [2], you can also use the hints in the back.
4. You can also use any other online or print references, e.g. searching the web. You may also ask other people for aid. However, I ask that you **reference any "outside sources"** that you used, for each problem, other than those in item 3.
5. Try to write your solutions clearly and completely; this matters significantly for decisions. See [1] for some suggestions.

§4 Problems

These problems are a subset of those in the Year VI application (for Fall 2020). You can check the application there for some additional context and suggestions for reading.

Problem 1 (#1.50, IMO 2013)**.** Let ABC be an acute triangle with orthocenter H, and let W be a point on the side $\overline{BC}$, between B and C. The points M and N are the feet of the altitudes drawn from B and C, respectively. Suppose ω_1 is the circumcircle of triangle BWN and X is a point such that $\overline{WX}$ is a diameter of ω_1. Similarly, ω_2 is the circumcircle of triangle CWM and Y is a point such that $\overline{WY}$ is a diameter of ω_2. Show that the points X, Y, and H are collinear.

Problem 2 (#2.28, JMO 2012)**.** Given a triangle ABC, let P and Q be points on segments $\overline{AB}$ and $\overline{AC}$, respectively, such that $AP = AQ$. Let S and R be distinct points on segment $\overline{BC}$ such that S lies between B and R, $\angle BPS = \angle PRS$, and $\angle CQR = \angle QSR$. Prove that P, Q, R, S are concyclic.

Problem 3 (#2.35, IMO 2009)**.** Let ABC be a triangle with circumcenter O. The points P and Q are interior points of the sides CA and AB respectively. Let K, L M be the midpoints of $\overline{BP}$, $\overline{CQ}$, $\overline{PQ}$, respectively, and let Γ be the circumcircle of $\triangle KLM$. Suppose that $\overline{PQ}$ is tangent to Γ. Prove that $OP = OQ$.

Problem 4 (#4.48, Japan 2009)**.** Triangle ABC has circumcircle Γ. A circle with center O is tangent to BC at P and internally to Γ at Q, so that Q lies on arc BC of Γ not containing A. Prove that if $\angle BAO = \angle CAO$ then $\angle PAO = \angle QAO$.

Problem 5. Suppose that $a^2 + b^2 + c^2 = 1$ for positive real numbers a, b, c. Find the minimum possible value of

$$\frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b}.$$

Problem 6. Let a, b, c be positive real numbers such that $a^2 + b^2 + c^2 + (a + b + c)^2 \le 4$. Prove that

$$\frac{ab+1}{(a+b)^2} + \frac{bc+1}{(b+c)^2} + \frac{ca+1}{(c+a)^2} \ge 3.$$

Problem 7. For positive real numbers a, b, c satisfying $abc = 1$, prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \ge \frac{3}{2}.$$

Problem 8. Find all functions $f \colon \mathbb{R} \to \mathbb{R}$ for which

$$f(xf(x) + f(y)) = f(x)^2 + y$$

holds for all real numbers x and y.

Problem 9. Ana and Banana are playing a game. First Ana picks a word, which is defined to be a nonempty sequence of capital English letters. Then Banana picks a nonnegative integer k and challenges Ana to supply a word with exactly k subsequences which are equal to Ana's word. Ana wins if she is able to supply such a word, otherwise she loses. For example, if Ana picks the word "TST", and Banana chooses $k = 4$, then Ana can supply the word "TSTST" which has 4 subsequences which are equal to Ana's word. Which words can Ana pick so that she can win no matter what value of k Banana chooses?

References

[1] Evan Chen. Remarks on English. http://web.evanchen.cc/handouts/english/english.pdf.

[2] Evan Chen. *Euclidean geometry in mathematical olympiads*. MAA Problem Books Series. Mathematical Association of America, Washington, DC, 2016.