OTIS Mock Olympiad

Exam Sample 08 JMO

Mock JMO (4 hours)

JMO 4. Solve $f(m+n) = f(m) + f(n) + mn$ for $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$.

JMO 5. Let $ABCD$ be a convex quadrilateral. Assume that the incircle of triangle ABD is tangent to $\overline{AB}$, $\overline{AD}$, $\overline{BD}$ at points W, Z, K. Also assume that the incircle of triangle CBD is tangent to $\overline{CB}$, $\overline{CD}$, $\overline{BD}$ at points X, Y, K. Prove that quadrilateral $WXYZ$ is cyclic.

JMO 6. Positive integers x_1, x_2, $\ldots$, x_n $(n \geq 4)$ are arranged in a circle such that each x_i divides the sum of the neighbors; that is,

$$\frac{x_{i-1} + x_{i+1}}{x_i} = k_i$$

is an integer for each i, where $x_0 = x_n$, $x_{n+1} = x_1$. Prove that

$$2 \leq \frac{k_1 + \cdots + k_n}{n} < 3.$$

Time limit: 4 hours
Each problem is worth 7 points