# OTIS Application Homework

## Year XII — OTIS 2026-2027

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https://apply.evanchen.cc

This packet contains instructions and the homework questions for students who wish to join OTIS. The actual application portal is at https://apply.evanchen.cc.

## §1 Philosophy: this is not a test

The application problems are treated differently from what you may be used to. Many programs have an "entrance exam" or "qualifying test" or similar, which serve as an exam, and the highest scores get in. This is *not* the intention of the OTIS application; which is not meant to compare applicants. I like the FAQ from the Euler Circle, which I've adapted for OTIS:

> The traditional way admissions are competitive is there are $N$ applicants and $M \ll N$ spaces available. That is NOT the case for OTIS, where $M > N$ at present. We dislike the idea of depriving students of an education; we will certainly not take any pride in doing so, and will never release statistics.

Instead, the OTIS application problems are meant mostly for a different purpose:

- It implicitly serves as your summer reading. Part I of my geometry book is basically a pre-requisite for OTIS (if you're doing high-school competitions), so working through the geometry problems in section A will check that you actually know the background.
- It lets me see your writing. If I have an easy time reading and understanding your solutions, that's usually a good sign that OTIS will work well. (This means it's to your advantage to write up solutions well!)
- It gives you some practice asking for help (see item 4 in instructions).
- It helps serve as a sanity check that you will have enough time to work on OTIS during the year. This packet consists of actual olympiad-level problems, so you can see what you are getting yourself into.

So, **please treat this like homework rather than a test**. In particular, you can even ask me for help on the problems (see item 4 in instructions). I will not just grade out of 7 and sum the scores (in fact, I probably won't even bother assigning scores). Instead, I am looking to see whether you are someone who is willing and able to solve olympiad problems and take the time to write them up cleanly.

Don't be discouraged if you find the problems challenging! If you start early, work diligently, and are willing to ask for hints, then I think you're likely to do well.

## §2 Instructions on solving

1. **Returners**: If you attended OTIS or MOP in any previous year, you do not have to do the problems. Skip to the next section (and ignore the PDF upload).

   **Newcomers**: Try to solve as many problems as you can.[1]

   - If you've finished high school and aren't competing in high-school contests, you can skip the geometry problems in Section A (but mention this explicitly).

2. **You must write the solutions yourself**; don't copy-paste someone else's work.

3. If you have seen a problem before, you may write any solution you remember, or use any solution that you've written yourself in the past.

4. **You can ask me for help if you're stuck on something!** Just send me an email telling me what you've tried, and I'll try to push you in the right direction.[2] This is how OTIS works for admitted students, so why not practice now?

   For the geometry problems, you can also use the hints in the back of the textbook.

5. You can also use any other online or print references, e.g. searching the web. You may also ask other people for aid. However, I ask that you **reference any "outside sources"** that you used, for each problem (see Problem C.5). This does not count negatively at all.

6. **You may not use generative AI or large language models** including but not limited to ChatGPT, Claude, Gemini, Grok, etc. (This especially applies to the coding problem, Problem C.1.)

7. Try to write your solutions clearly and completely; I care about writing. See https://web.evanchen.cc/handouts/english/english.pdf for some suggestions.

## §3 Submission instructions on apply.evanchen.cc

8. To participate in the full year of OTIS, apply by August 1, 2026. Applications submitted by then will be processed no later than end of August.

9. You can continue to submit late applications up until April 30, 2027.[3] (The cost is still $240 per semester; joining late does not decrease the cost. That is, the cost is not pro-rated, even if you join on the last day.)

10. Submit applications via https://apply.evanchen.cc. This form will also contain some questions for you to fill out, such as your grade level and background. Financial aid requests are also submitted through this portal.

    - You will need either a Google or GitHub account. If you can't get either, email me and I'll manually create a login for you.

[1] Occasionally people ask me for a cutoff number of problems. I won't answer because I think you should try to sweep all the problems rather than doing the minimum work and stopping. If you are bartering to do less before OTIS has even started, this is really the wrong program for you.

[2] Warning: I travel for IMO over the summer, so responses then will be slower. Start early.

[3] If you send a late application before September 1, 2026, it could take a while to process because August is a busy month for OTIS.

11. Most homework solutions should be submitted as a **single PDF**, which is not to exceed **10 megabytes**. Scans might exceed that limit, so LaTeX is encouraged. If you want to handwrite, then learn how to compress your PDF submissions.

12. A few of the homework problems should be submitted **directly on the website instead of in your submission PDF**. These five problems are marked with the prefix [Submit on web form] to indicate this. They are:
    - The coding problem Problem C.1 has a space for your answer and a space for your source code.
    - The attestation problem Problem C.5 has a text area.
    - The three reading comprehension questions in Section D.

13. Application decisions and comments are posted directly on the portal.
    - If you apply by August 1, 2026, you will get a decision by the end of August, though possibly sooner. (For example, in 2025-2026 I actually got decisions out within a week.)
    - If you apply after September 1, 2026, you can usually expect decisions within 1-3 days. If it's been more than 3 days, you can email me to check in.

## §4 Meme for your amusement

# Homework problems

## §A Geometry homework problems from my textbook

- **Reading**: It's suggested to read Part I of my book as the material there is necessary (and sufficient) to solve these problems.
- All problems are themselves from the first three chapters of my geometry book and you can use the hints provided.
- It is not essential that you typeset diagrams for these problems.
- All problems admit synthetic solutions, but computational approaches are okay too. Do whatever you need to.
- If you have completed high school and are not competing in high-school contests, then you may choose to skip this section (but state this explicitly).

**Problem A.1** (#2.28, JMO 2012)**.** Given a triangle $ABC$, let $P$ and $Q$ be points on segments $\overline{AB}$ and $\overline{AC}$, respectively, such that $AP = AQ$. Let $S$ and $R$ be distinct points on segment $\overline{BC}$ such that $S$ lies between $B$ and $R$, $\angle BPS = \angle PRS$, and $\angle CQR = \angle QSR$. Prove that $P$, $Q$, $R$, $S$ are concyclic.

**Problem A.2** (#2.35, IMO 2009)**.** Let $ABC$ be a triangle with circumcenter $O$. The points $P$ and $Q$ are interior points of the sides $CA$ and $AB$ respectively. Let $K$, $L$, $M$ be the midpoints of $\overline{BP}$, $\overline{CQ}$, $\overline{PQ}$. Suppose that $\overline{PQ}$ is tangent to the circumcircle of $\triangle KLM$. Prove that $OP = OQ$.

**Problem A.3** (#3.25, USAMO 1993)**.** Let $ABCD$ be a quadrilateral whose diagonals are perpendicular and meet at $E$. Prove that the reflections of $E$ across the sides of $ABCD$ are concyclic.

## §B Inequalities homework problems

- **Reading**: You should read §2.1, §2.2, §2.4 of The OTIS Excerpts as the material there is necessary (and sufficient) to solve these problems.

**Problem B.1.** Suppose that $a^2 + b^2 + c^2 = 1$ for positive real numbers $a$, $b$, $c$. Find the minimum possible value of

$$\frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b}.$$

**Problem B.2.** Let $a$, $b$, $c$ be positive real numbers such that $a^2+b^2+c^2+(a+b+c)^2 \le 4$. Prove that

$$\frac{ab+1}{(a+b)^2} + \frac{bc+1}{(b+c)^2} + \frac{ca+1}{(c+a)^2} \ge 3.$$

**Problem B.3.** Let $a$, $b$, $c$, $d$ be positive reals with $(a+c)(b+d) = 1$. Prove that

$$\frac{a^3}{b+c+d} + \frac{b^3}{c+d+a} + \frac{c^3}{d+a+b} + \frac{d^3}{a+b+c} \ge \frac{1}{3}.$$

# §C Additional homework problems

- **Reading**: If you don't know how to code, I suggest reading up to §4.4 of https://docs.python.org/3/tutorial/index.html. If you are new to functional equations, read Chapter 3 of https://web.evanchen.cc/excerpts.html.

**Problem C.1** (Learn to code, please, I implore you, for the love of god, I am so sick of people who just say "idk code" and adamantly refuse to learn, please grow a spine)**.** [Submit on web form] Write a computer program to find the number of ordered pairs of prime numbers $(p, q)$ such that when

$$N = p^2 + q^3$$

is written in decimal (without leading zeros), each digit from 0 to 9 appears exactly once. For example, $(109, 1163)$ is one such pair because $109^2 + 1163^3 = 1573049628$.

You should write the code yourself — again, AI is explicitly not allowed (on the entire homework).

> **Remark.** Some tips for the programming problem:
>
> - If you use C/C++, be careful of integer overflow.
> - As a confirmation, the correct answer is 10 times a prime number.
> - As a benchmark, an Evan-level Python 3 programmer can easily solve this problem in 15 lines, without any external imports. If you're a newbie, your program will probably be a lot longer; you don't need to spend too much time optimizing.

**Problem C.2.** Find all functions $f\colon \mathbb{R} \to \mathbb{R}$ for which

$$f(xf(x) + f(y)) = f(x)^2 + y$$

holds for all real numbers $x$ and $y$.

**Problem C.3.** Let $a$, $b$, $c$, $d$ be real numbers such that $b - d \geq 5$ and all zeros $x_1$, $x_2$, $x_3$, and $x_4$ of the polynomial $P(x) = x^4 + ax^3 + bx^2 + cx + d$ are real. Find the smallest value the product $(x_1^2 + 1)(x_2^2 + 1)(x_3^2 + 1)(x_4^2 + 1)$ can take.

**Problem C.4.** Ana and Banana are playing a game. First Ana picks a word, which is defined to be a nonempty sequence of capital English letters. Then Banana picks a nonnegative integer $k$ and challenges Ana to supply a word with exactly $k$ subsequences which are equal to Ana's word. Ana wins if she is able to supply such a word, otherwise she loses. For example, if Ana picks the word "TST", and Banana chooses $k = 4$, then Ana can supply the word "TSTST" which has 4 subsequences which are equal to Ana's word. Which words can Ana pick so that she can win no matter what value of $k$ Banana chooses?

**Problem C.5** (Honor code)**.** [Submit on web form] Truthfully copy the following statement on the application portal, replacing *(list of sources)* with a list of outside sources you used. (Write "none" if no sources were used.)

> *I assert that all writing submitted is my own and that I did not use any AI or large language models. The sources I used in completing this homework were: (list of sources).*

## §D Reading comprehension

This is designed to make sure you actually read the entire OTIS syllabus before you submit a complete application.

**Problem D.1.** [Submit on web form] What was the answer in the example outline?

**Problem D.2.** [Submit on web form] If you have $500\clubsuit + 300\heartsuit + 75\spadesuit + 40\diamondsuit$ then what level are you?

**Problem D.3.** [Submit on web form] How many of the ten practice exams are graded?

As a check, if $x, y, z \in \mathbb{Z}$ are the three answers, then $x + y + z$ is prime.

## §E Extra credit (for fun, not considered for admissions)

This section is only if you're really bored over the summer or something and are looking to pick up new hobbies.

**Problem E.1.** Which Dreamcatcher song is the best?

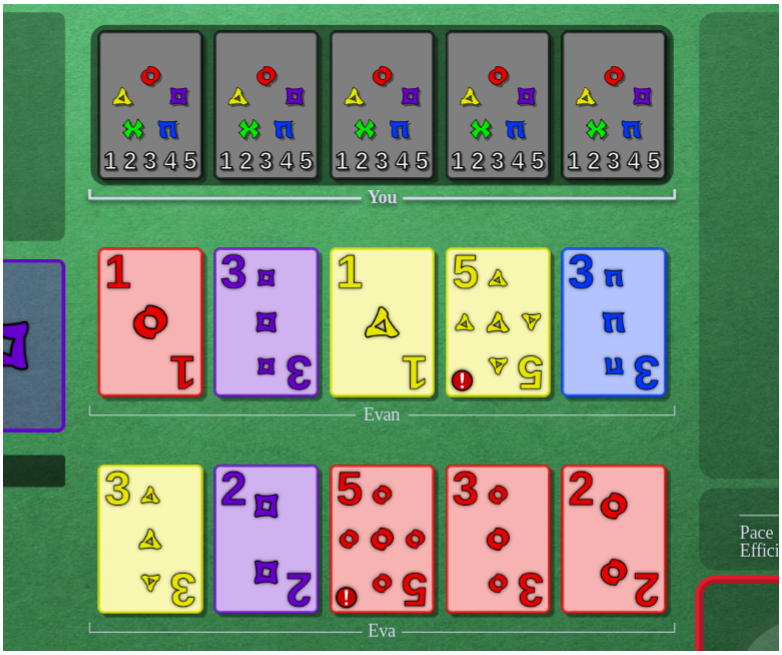
**Problem E.2.** Solve as many puzzles from the MOSP 2021 hunt as you can.

**Problem E.3.** You're playing a beginner game of Hanabi with Evan and Eva using the conventions listed in https://tinyurl.com/hanabi-evan-intro. It's the first turn and the game looks as shown below.

(a) What are the legal moves under this convention set?

(b) Of these, which one do you think is the best and why?

If you play with higher-level conventions, you can answer for those instead.




Full image: https://web.evanchen.cc/upload/hanabi243.png