Notes for 18.02 Recitation 13

18.02 Recitation MW9

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The world is quiet here.

— *Lemony Snicket, in A Series of Unfortunate Events*

This handout (and any other DLC's I write) are posted at https://web.evanchen.cc/1802.html. (This was originally titled Recitation 14, but the numbering changed.)

§1 Reading

You should read section 21 of LAMV for the full details, which don't fit on the page. The following are just excerpts for quick reference.

§2 Recipes

Recipe for integrating over a rectangle

To integrate something of the form $\int \left(\int \, \mathrm{d}y \right) \mathrm{d}x$:

1. Evaluate the inner integral as in 18.01, treating x as constant.
2. You should get something only depending on x. Integrate it as in 18.01.

Recipe for converting to xy-integration

1. Draw a picture of the region as best you can.
2. Write the region as a list of inequalities.[1]
3. Pick *one* of x and y, and use your picture to describe all the values it could take.
4. Solve for the *other* variable in all the inequalities.

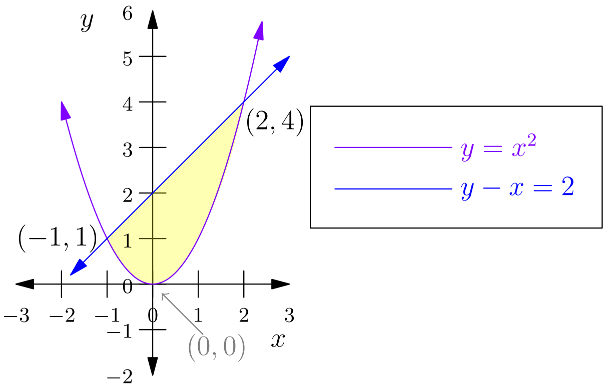
§3 Pictures for the example from Poonen's notes

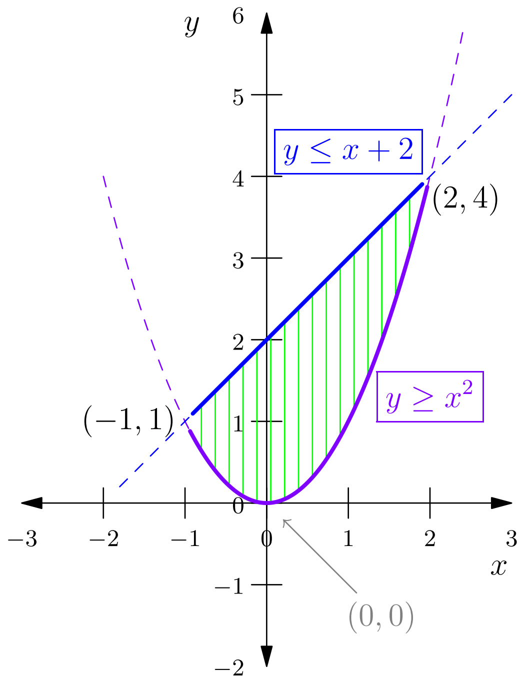
Sample Question

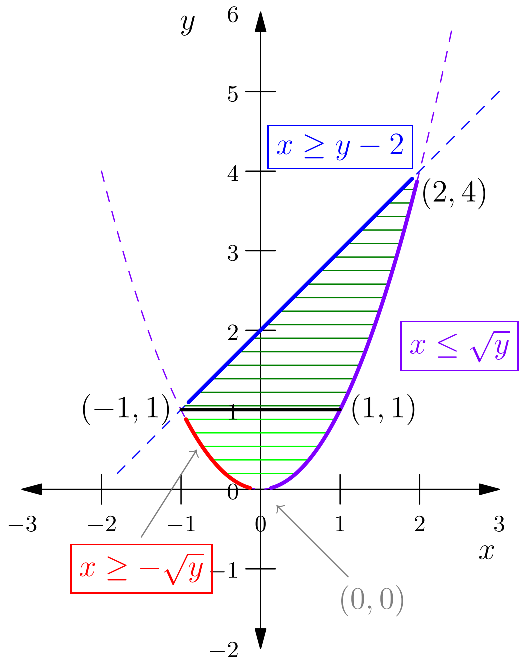
Show both ways of setting up an integral of a function $f(x, y)$ over the region bounded by $y - x = 2$ and $y = x^2$.

Here the region would be described as $y \geq x^2$ and $y - x \leq 2$.

[1] I don't think other sources always write the inequalities the way I do. But I think this will help you a lot with making sure bounds go the right way.







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§4 Recitation questions from official course

1. Calculate the double integral of the function $f(x, y) = 6x^2 + 2y$ over the rectangle $R = [0, 2] \times [-1, 1]$. Use both vertical and horizontal slicings and check you get the same answer.

2. Let R be the first-quadrant region bounded by the two curves $y = \sqrt{x}$ and $y = x^3$. Compute in two different ways the double integral

$$\iint_R xy^2 \, \mathrm{d}A.$$

3. Let R be the (bounded) region between the parabola $y^2 = x$ and the line through $(2, 0)$ having slope 1. Find the points where the curves intersect and describe the region R in terms of horizontal slices and vertical slices. Express the double integral $\iint_R f(x, y) \, \mathrm{d}A$ as an iterated integral in both ways, using both horizontal and vertical slicings. In the second case, you will have to write the integral in two pieces.