

# Notes for 18.02 Recitation 10

## 18.02 Recitation MW9

EVAN CHEN

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*There is  
one art,  
no more,  
no less:  
to do  
all things  
with art-  
lessness.*

— *Ars Brevis, by Piet Hein*

This handout (and any other DLC's I write) are posted at <https://web.evanchen.cc/1802.html>.

### I'm writing unified course notes now!

I'm working on writing a replacement set of lecture notes! You can find them posted on my usual website, filename `lamv.pdf`.

### Say hi to Lichen

At some point (I don't know exactly when) Lichen will be here as a teaching mentee, meaning he'll observe some number of the recitations and then try to teach one (and get feedback from you all). Please be nice to Lichen!

## §1 Review of Oct 2 recitation and Oct 3-4 lecture

I won't repeat myself from `r09.pdf`; I have about 10 extra copies of `r09.pdf` if you need one.

### §1.1 Level curves replace $xy$ -graphs from high school

You'll mostly want to work with **level-curve pictures**, in which case

- all the variables  $x, y, z$  etc. are inputs
- the output  $f(x, y)$  is **unnamed**; no variable name for it. (Sometimes just  $f$  if really needed, like in the last question on PSet 4B, but usually it's anonymous.)

Contour plot is a synonym for level curve, if you run into that term.

### §1.2 Type signatures for this section

(If you're new to the recitation, read the handout from recitation 1.) Relevant types:

- $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is a function that accepts *points* and outputs *numbers*.
- $\nabla f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a function that accepts *points* and outputs *vectors*.

Although points and vectors were used interchangeably before, it might help to think of them as different types for this part of the course, because you will draw and use them rather differently. (For example, we don't ever add two points.)

- A *point* represents a position in your input space  $\mathbb{R}^n$ . You should **draw a point as a dot** (rather than arrows). I'll try to use capital letters  $P, Q, \dots$  for these.
- A *vector* is usually a displacement of some sort in this part of the course; so you should always **draw vectors as arrows**. I'll try to use bold lowercase letters  $\mathbf{u}, \mathbf{v}, \dots$  for these.

### §1.3 Gradient replaces derivative

See the table in r09.pdf. The two important things to know are:

- Linear approximation:  $f(P + \mathbf{u}) \approx f(P) + \nabla f \cdot \mathbf{u}$
- $\nabla f$  is a normal vector to the tangent of the level curve at  $P$ .

To concretely compute  $\nabla f$ , take the partial derivatives of  $f$  with respect to each of the input variables.

### §1.4 Notational difference

I think Maulik places arrows over  $\nabla f$  to emphasize it's a vector, but I'm not going to write the arrow because I think it makes the equation harder to read. Feel free to add the arrow yourself if it helps you, though.

## §2 Recitation questions from official course

- 1 Consider the function  $f(x, y) = \frac{1}{x^2 + y^2}$ .
  1. Draw the level curves for  $f(x, y) = 1$  or  $2$ .
  2. Sketch the graph of  $f$ .
  3. Find the partial derivatives  $f_x(x, y)$  and  $f_y(x, y)$ .
  4. Starting at the point  $(1, 2)$ , in what unit vector direction  $\mathbf{u}$  does  $f(x, y)$  increase the fastest? What is the rate of increase in this direction?
  5. What is the directional derivative  $D_{\mathbf{u}}f(1, 2)$  where  $\mathbf{u}$  is the direction given by the unit vector  $\langle 1/\sqrt{5}, -2/\sqrt{5} \rangle$ ?
- 2 Estimate the value of  $\log(0.49^2 + 0.76)$  by calculating the linear approximation of the function  $\log(x^2 + y)$  near the point  $(0.5, 0.75)$ . (Here I'm using  $\log$  for natural logarithm).
- 3 Consider the surface  $S$  given by the equation  $xy - 2xz^2 + 3y^2z = 2$ . If I consider a vector  $\mathbf{u}$  that is tangent to  $S$  at the point  $(1, 1, 1)$ , what is the directional derivative of  $f(x, y, z) = xy - 2xz^2 + 3y^2z$  at  $(1, 1, 1)$  in direction  $\mathbf{u}$ ? What is the equation of the tangent plane to  $S$  at the point  $(1, 1, 1)$ ?