Quick answer key to Recitation 7 (Exam 1 practice)

ChatGPT 4o

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I just used ChatGPT to write this one quickly. ChatGPT can make mistakes, so if you spot anything that's wrong, flag me to ask.

Contents

1 Solution

We are given the points $P = (1, 0, 1)$, $Q = (1, 1, 2)$, and $R = (-1, 1, 1)$. We will compute the required values step by step.

1.1 Part 1: Vector from P to the midpoint of the line segment connecting Q and R

The midpoint M of the line segment connecting Q and R is given by the average of the coordinates of Q and R:

$$M = \left(\frac{1 + (-1)}{2}, \frac{1 + 1}{2}, \frac{2 + 1}{2} \right) = (0, 1, \frac{3}{2})$$

The vector connecting P to M is the difference between their coordinates:

$$\overrightarrow{PM} = M - P = (0 - 1, 1 - 0, \frac{3}{2} - 1) = (-1, 1, \frac{1}{2})$$

Thus, the vector from P to the midpoint of the line segment connecting Q and R is:

$$\overrightarrow{PM} = (-1, 1, \frac{1}{2})$$

1.2 Part 2: Area of the triangle with vertices P, Q, R

The area of the triangle with vertices P, Q, R is given by:

$$\text{Area} = \frac{1}{2} \left\| \overrightarrow{PQ} \times \overrightarrow{PR} \right\|$$

First, we compute the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$:

$$\overrightarrow{PQ} = Q - P = (1-1, 1-0, 2-1) = (0, 1, 1)$$

$$\overrightarrow{PR} = R - P = (-1-1, 1-0, 1-1) = (-2, 1, 0)$$

Next, we compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$:

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ -2 & 1 & 0 \end{vmatrix} = \mathbf{i} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix}$$

$$= \mathbf{i}(1(0) - 1(1)) - \mathbf{j}(0(0) - 1(-2)) + \mathbf{k}(0(1) - 1(-2))$$

$$= \mathbf{i}(-1) - \mathbf{j}(2) + \mathbf{k}(2)$$

$$= (-1, -2, 2)$$

Now, compute the magnitude of this vector:

$$\left\| \overrightarrow{PQ} \times \overrightarrow{PR} \right\| = \sqrt{(-1)^2 + (-2)^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$$

Thus, the area of the triangle is:

$$\text{Area} = \frac{1}{2} \times 3 = \frac{3}{2}$$

1.3 Part 3: Equation of the plane through P, Q, R

The normal vector to the plane is given by the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$, which we found to be $(-1, -2, 2)$. The equation of a plane passing through a point (x_0, y_0, z_0) with normal vector $\mathbf{n} = \langle A, B, C \rangle$ is:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

Substitute $A = -1$, $B = -2$, $C = 2$, and the coordinates of $P = (1, 0, 1)$:

$$-1(x - 1) - 2(y - 0) + 2(z - 1) = 0$$

Simplifying:

$$-(x - 1) - 2y + 2(z - 1) = 0$$

$$-x + 1 - 2y + 2z - 2 = 0$$

$$-x - 2y + 2z - 1 = 0$$

Thus, the equation of the plane is:

$$x + 2y - 2z = -1$$

2 Solution

We are given the following conditions:

$$x_1^2 + x_2^2 + x_3^2 = 4 \quad \text{and} \quad y_1^2 + y_2^2 + y_3^2 = 9$$

We are asked to find the range of possible values for:

$$x_1y_1 + x_2y_2 + x_3y_3$$

This expression is the dot product of the vectors $\mathbf{x} = \langle x_1, x_2, x_3 \rangle$ and $\mathbf{y} = \langle y_1, y_2, y_3 \rangle$.

Step 1: Use the dot product formula

The dot product of two vectors $\mathbf{x}$ and $\mathbf{y}$ is given by:

$$\mathbf{x} \cdot \mathbf{y} = x_1y_1 + x_2y_2 + x_3y_3 = |\mathbf{x}||\mathbf{y}| \cos\theta$$

where:

$$|\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2} = \sqrt{4} = 2$$
$$|\mathbf{y}| = \sqrt{y_1^2 + y_2^2 + y_3^2} = \sqrt{9} = 3$$

and θ is the angle between the vectors $\mathbf{x}$ and $\mathbf{y}$.

Thus, the dot product becomes:

$$\mathbf{x} \cdot \mathbf{y} = 2 \cdot 3 \cdot \cos\theta = 6\cos\theta$$

Step 2: Determine the range of values

Since $\cos\theta$ ranges between -1 and 1, the dot product $\mathbf{x}\cdot\mathbf{y}$ will range between:

$$6\cos\theta \quad \text{where} \quad -1 \leq \cos\theta \leq 1$$

Thus, the range of $\mathbf{x} \cdot \mathbf{y}$ is:

$$-6 \leq x_1y_1 + x_2y_2 + x_3y_3 \leq 6$$

Conclusion

The range of possible values for $x_1y_1 + x_2y_2 + x_3y_3$ is:

$$\boxed{[-6, 6]}$$

3 Solution

We are given the planes:

$$P_1 : x + 2y + 3z = 0 \quad \text{and} \quad P_2 : 2y - z = 0$$

3.1 Part 1: A vector parallel to both P_1 and P_2

To find a vector parallel to both planes, we note that the normal vector to a plane is perpendicular to all vectors lying in the plane. The normal vectors to the planes are:

$$\mathbf{n}_1 = \langle 1, 2, 3 \rangle \quad \text{for} \quad P_1$$

$$\mathbf{n}_2 = \langle 0, 2, -1 \rangle \quad \text{for} \quad P_2$$

A vector that is parallel to both planes must be perpendicular to both normal vectors. Such a vector can be found by taking the cross product of $\mathbf{n}_1$ and $\mathbf{n}_2$.

We compute the cross product $\mathbf{n}_1 \times \mathbf{n}_2$:

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 0 & 2 & -1 \end{vmatrix} = \mathbf{i} \begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix}$$

$$= \mathbf{i}\left((2)(-1) - (3)(2)\right) - \mathbf{j}\left((1)(-1) - (3)(0)\right) + \mathbf{k}\left((1)(2) - (2)(0)\right)$$

$$= \mathbf{i}(-2 - 6) - \mathbf{j}(-1) + \mathbf{k}(2) = -8\mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

$$= \langle -8, 1, 2 \rangle$$

Thus, a vector parallel to both planes is:

$$\langle -8, 1, 2 \rangle$$

3.2 Part 2: Distance from the point $(2, 1, 4)$ to the plane P_1

The formula for the distance from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is given by:

$$\text{Distance} = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

For the plane $P_1 : x + 2y + 3z = 0$, we have:

$$A = 1, \quad B = 2, \quad C = 3, \quad D = 0$$

The point is $(2, 1, 4)$, so we substitute the values into the formula:

$$\text{Distance} = \frac{|1(2) + 2(1) + 3(4) + 0|}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{|2 + 2 + 12|}{\sqrt{1 + 4 + 9}} = \frac{16}{\sqrt{14}}$$

$$\text{Distance} = \frac{16}{\sqrt{14}} \approx 4.28$$

Thus, the distance from the point $(2, 1, 4)$ to the plane P_1 is $\frac{16}{\sqrt{14}} \approx 4.28$.

4 Solution

4.1 Part 1: Rotation matrix M associated with counterclockwise rotation by $\frac{5\pi}{4}$

The matrix associated with a counterclockwise rotation by an angle θ in $\mathbb{R}^2$ is given by:

$$M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

In this case, the angle of rotation is $\theta = \frac{5\pi}{4}$.

First, we compute $\cos\left(\frac{5\pi}{4}\right)$ and $\sin\left(\frac{5\pi}{4}\right)$:

$$\cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}, \quad \sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

Thus, the rotation matrix M is:

$$M = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

4.2 Part 2: Calculate MN and NM

We are given the matrix $N = \begin{pmatrix} 1 & 2 & 4 \\ -3 & 6 & 2 \end{pmatrix}$.

Check if MN is defined

The matrix M is a 2×2 matrix, and N is a 2×3 matrix. The product MN is defined because the number of columns in M matches the number of rows in N. The result will be a 2×3 matrix.

We now calculate MN:

$$MN = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ -3 & 6 & 2 \end{pmatrix}$$

We compute each element of the resulting matrix by taking the dot product of the rows of M with the columns of N:

$$MN = \begin{pmatrix} \left(-\frac{1}{\sqrt{2}}(1) + \frac{1}{\sqrt{2}}(-3)\right) & \left(-\frac{1}{\sqrt{2}}(2) + \frac{1}{\sqrt{2}}(6)\right) & \left(-\frac{1}{\sqrt{2}}(4) + \frac{1}{\sqrt{2}}(2)\right) \\ \left(-\frac{1}{\sqrt{2}}(1) + -\frac{1}{\sqrt{2}}(-3)\right) & \left(-\frac{1}{\sqrt{2}}(2) + -\frac{1}{\sqrt{2}}(6)\right) & \left(-\frac{1}{\sqrt{2}}(4) + -\frac{1}{\sqrt{2}}(2)\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-1+3}{\sqrt{2}} & \frac{-2+6}{\sqrt{2}} & \frac{-4+2}{\sqrt{2}} \\ \frac{-1-3}{\sqrt{2}} & \frac{-2-6}{\sqrt{2}} & \frac{-4-2}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{2}} & \frac{4}{\sqrt{2}} & \frac{-2}{\sqrt{2}} \\ \frac{-4}{\sqrt{2}} & \frac{-8}{\sqrt{2}} & \frac{-6}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{2} & 2\sqrt{2} & -\sqrt{2} \\ -2\sqrt{2} & -4\sqrt{2} & -3\sqrt{2} \end{pmatrix}$$

Check if NM is defined

The matrix N is 2×3 and M is 2×2. The product NM is not defined because the number of columns in N does not match the number of rows in M.

Thus, NM is **not defined**.

Conclusion

The matrix product MN is:

$$MN = \begin{pmatrix} \sqrt{2} & 2\sqrt{2} & -\sqrt{2} \\ -2\sqrt{2} & -4\sqrt{2} & -3\sqrt{2} \end{pmatrix}$$

The matrix product NM is not defined.

5 Solution

We are given the vectors:

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

5.1 Part 1: Volume of the parallelepiped

Take the determinant:

$$\det \begin{bmatrix} 2 & 3 & 0 \\ 0 & 5 & 1 \\ 1 & 2 & 0 \end{bmatrix} = -1.$$

Thus, the volume of the parallelepiped is:

$$\text{Volume} = |-1| = 1$$

5.2 Part 2: Are these vectors a basis for $\mathbb{R}^3$?

To determine if the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a basis for $\mathbb{R}^3$, we check if they are linearly independent. The vectors are linearly independent if the determinant above was non-zero.

Since the determinant is -1, which is non-zero, the vectors are linearly independent.

Therefore, the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a basis for $\mathbb{R}^3$.

6 Solution

We are given the system of equations:

$$x + 3y = 0 \quad (1)$$

$$-ax - y = 1 \quad (2)$$

6.1 Part 1: Matrix equation

We can write this system of equations as a matrix equation. The system can be written as:

$$\begin{pmatrix} 1 & 3 \\ -a & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Thus, the matrix equation is:

$$A\mathbf{v} = \mathbf{b}$$

where:

$$A = \begin{pmatrix} 1 & 3 \\ -a & -1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

6.2 Part 2: Values of a for which the system has a unique solution

The system has a unique solution if the matrix A is invertible, which occurs when the determinant of A is non-zero. The determinant of A is given by:

$$\det(A) = \det \begin{pmatrix} 1 & 3 \\ -a & -1 \end{pmatrix} = (1)(-1) - (3)(-a) = -1 + 3a$$

For the matrix to be invertible, we require $\det(A) \neq 0$:

$$-1 + 3a \neq 0$$

$$3a \neq 1 \quad \Rightarrow \quad a \neq \frac{1}{3}$$

Thus, the system has a unique solution for all values of a except $a = \frac{1}{3}$.

6.3 Part 3: Solution in terms of a using the inverse matrix

To solve the system using the inverse of the matrix, we first compute the inverse of A, assuming $a \neq \frac{1}{3}$.

The inverse of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

For the matrix $A = \begin{pmatrix} 1 & 3 \\ -a & -1 \end{pmatrix}$, we already know that:

$$\det(A) = -1 + 3a$$

Thus, the inverse of A is:

$$A^{-1} = \frac{1}{-1+3a} \begin{pmatrix} -1 & -3 \\ a & 1 \end{pmatrix}$$

Now, we solve for $\mathbf{v} = A^{-1}\mathbf{b}$:

$$\begin{aligned} \mathbf{v} &= \frac{1}{-1+3a} \begin{pmatrix} -1 & -3 \\ a & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \frac{1}{-1+3a} \begin{pmatrix} -3 \\ 1 \end{pmatrix} \end{aligned}$$

Thus, the solution is:

$$\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-1+3a} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

The solutions for x and y in terms of a are:

$$x = \frac{-3}{-1+3a}, \quad y = \frac{1}{-1+3a}$$

7 Solution

We are given the matrix:

$$A = \begin{pmatrix} 5 & 8 \\ 7 & 4 \end{pmatrix}$$

7.1 Part 1: Characteristic polynomial and eigenvalues

The characteristic polynomial of a matrix A is given by:

$$\det(A - \lambda I) = 0$$

where λ is an eigenvalue and I is the identity matrix.

We compute $A - \lambda I$:

$$A - \lambda I = \begin{pmatrix} 5 & 8 \\ 7 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 - \lambda & 8 \\ 7 & 4 - \lambda \end{pmatrix}$$

Now, we compute the determinant of $A - \lambda I$:

$$\det(A - \lambda I) = \det \begin{pmatrix} 5 - \lambda & 8 \\ 7 & 4 - \lambda \end{pmatrix}$$

$$= (5 - \lambda)(4 - \lambda) - (8)(7)$$

$$= (5 - \lambda)(4 - \lambda) - 56$$

$$= 20 - 9\lambda + \lambda^2 - 56 = \lambda^2 - 9\lambda - 36$$

Thus, the characteristic polynomial is:

$$\lambda^2 - 9\lambda - 36 = 0$$

We solve this quadratic equation using the quadratic formula:

$$\lambda = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-36)}}{2(1)} = \frac{9 \pm \sqrt{81 + 144}}{2} = \frac{9 \pm \sqrt{225}}{2} = \frac{9 \pm 15}{2}$$

The solutions are:

$$\lambda_1 = \frac{9 + 15}{2} = 12, \quad \lambda_2 = \frac{9 - 15}{2} = -3$$

Thus, the eigenvalues of A are:

$$\lambda_1 = 12, \quad \lambda_2 = -3$$

7.2 Part 2: Eigenvector corresponding to the largest eigenvalue $\lambda_1 = 12$

To find the eigenvector corresponding to $\lambda_1 = 12$, we solve the system:

$$(A - 12I)\mathbf{v} = 0$$

where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$.

First, compute $A - 12I$:

$$A - 12I = \begin{pmatrix} 5-12 & 8 \\ 7 & 4-12 \end{pmatrix} = \begin{pmatrix} -7 & 8 \\ 7 & -8 \end{pmatrix}$$

Now, solve the system $(A - 12I)\mathbf{v} = 0$:

$$\begin{pmatrix} -7 & 8 \\ 7 & -8 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This gives the system of equations:

$$-7v_1 + 8v_2 = 0 \quad (1)$$

$$7v_1 - 8v_2 = 0 \quad (2)$$

Both equations are the same, so we can solve for v_1 in terms of v_2. From equation (1), we get:

$$-7v_1 + 8v_2 = 0 \quad \Rightarrow \quad v_1 = \frac{8}{7}v_2$$

Thus, a corresponding eigenvector is:

$$\mathbf{v} = \begin{pmatrix} \frac{8}{7}v_2 \\ v_2 \end{pmatrix} = v_2 \begin{pmatrix} \frac{8}{7} \\ 1 \end{pmatrix}$$

For simplicity, we can choose $v_2 = 7$, which gives:

$$\mathbf{v} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$$

Therefore, an eigenvector corresponding to $\lambda_1 = 12$ is:

$$\mathbf{v} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$$

8 Solution

8.1 Part 1: Calculate $(1 - i\sqrt{3})^7$ by converting to polar form

We are tasked with finding $(1 - i\sqrt{3})^7$. To do this, we first convert $1 - i\sqrt{3}$ to polar form and then apply De Moivre's theorem.

Step 1: Convert to polar form The modulus r of $1 - i\sqrt{3}$ is:

$$r = |1 - i\sqrt{3}| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

Next, we calculate the argument θ:

$$\theta = \arg(1 - i\sqrt{3}) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$$

Thus, the polar form of $1 - i\sqrt{3}$ is:

$$1 - i\sqrt{3} = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

Step 2: Apply De Moivre's theorem Using De Moivre's theorem, we compute:

$$\begin{aligned}(1 - i\sqrt{3})^7 &= \left[2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)\right]^7 \\ &= 2^7\left(\cos\left(7 \times -\frac{\pi}{3}\right) + i\sin\left(7 \times -\frac{\pi}{3}\right)\right) \\ &= 128\left(\cos\left(-\frac{7\pi}{3}\right) + i\sin\left(-\frac{7\pi}{3}\right)\right)\end{aligned}$$

Since $-\frac{7\pi}{3} = -2\pi - \frac{\pi}{3}$, we simplify using periodicity:

$$\cos\left(-\frac{7\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}, \quad \sin\left(-\frac{7\pi}{3}\right) = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Thus, we have:

$$\begin{aligned}(1 - i\sqrt{3})^7 &= 128\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) \\ &= 128 \times \frac{1}{2} + 128 \times \left(-\frac{\sqrt{3}}{2}\right)i \\ &= 64 - 64\sqrt{3}i\end{aligned}$$

Therefore:

$$(1 - i\sqrt{3})^7 = 64 - 64\sqrt{3}i$$

8.2 Part 2: Find zw and $\frac{z}{w}$

We are given $z = 2 + 3i$ and $w = 1 + 2i$.

Step 1: Compute zw The product of two complex numbers is given by:

$$zw = (2 + 3i)(1 + 2i)$$

Expand the product:

$$zw = 2(1) + 2(2i) + 3i(1) + 3i(2i)$$

$$= 2 + 4i + 3i + 6i^2$$

Since $i^2 = -1$, this simplifies to:

$$zw = 2 + 7i + 6(-1) = 2 + 7i - 6 = -4 + 7i$$

Thus:

$$zw = -4 + 7i$$

Step 2: Compute $\frac{z}{\overline{w}}$ The conjugate of $w = 1 + 2i$ is $\overline{w} = 1 - 2i$. We now compute $\frac{z}{\overline{w}}$:

$$\frac{z}{\overline{w}} = \frac{2 + 3i}{1 - 2i}$$

We multiply the numerator and denominator by the conjugate of the denominator $1 + 2i$:

$$\frac{z}{\overline{w}} = \frac{(2 + 3i)(1 + 2i)}{(1 - 2i)(1 + 2i)}$$

First, compute the denominator:

$$(1 - 2i)(1 + 2i) = 1^2 - (2i)^2 = 1 - (-4) = 5$$

Now, compute the numerator:

$$(2 + 3i)(1 + 2i) = 2(1) + 2(2i) + 3i(1) + 3i(2i)$$

$$= 2 + 4i + 3i + 6i^2 = 2 + 7i + 6(-1) = 2 + 7i - 6 = -4 + 7i$$

Thus:

$$\frac{z}{\overline{w}} = \frac{-4 + 7i}{5} = -\frac{4}{5} + \frac{7}{5}i$$

Therefore:

$$\frac{z}{\overline{w}} = -\frac{4}{5} + \frac{7}{5}i$$