JMO 2026/6

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Twitch Solves ISL

Episode 176

Problem

Emily has a red sheet of paper. She draws 2026 circles (not necessarily of equal size) on the piece of paper. She chooses a circle to color black, then cuts the paper around the circumference of all 2026 circles. She then separates the pieces of paper, into at least 2 black pieces and some number of red pieces. Is it possible that all black pieces are congruent?

Video

https://youtu.be/LZ4J5GQwrgA

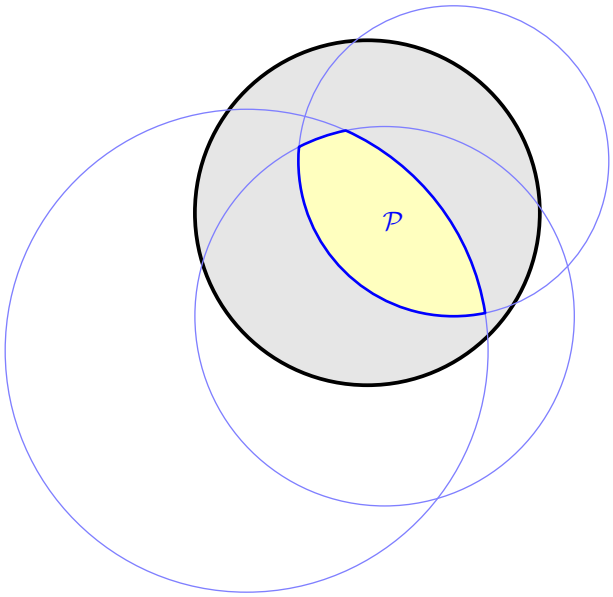
External Link

https://aops.com/community/p37586342

Solution

No, this is impossible.

Let Γ be the black circle, and for each point inside it consider the number of the 2026 circles it's contained in. Let X be a point for which that number achieves its maximal value, and consider the piece $\mathcal{P}$ containing X.



Claim. All the curves that bound $\mathcal{P}$ point outwards.

Proof. If not, then one could replace X with a point Y just across an inwards-pacing curve bounding $\mathcal{P}$. □

However, it's clear that if there are more than two black pieces, then some piece has a curve that points inwards. So it's impossible that the pieces are all congruent.