# JMO 2026/1

**Evan Chen**

Twitch Solves ISL

Episode 176

## Problem

Let $a$, $b$, $c$ be distinct positive integers such that $ab + c = c^2$. Prove that $(a - b)^2 \geq 4c$.

## External Link

https://aops.com/community/p37578110

## Solution

Note that the conclusion is equivalent to

$$(a-b)^2 \geq 4c \iff (a+b)^2 \geq 4c + 4ab = 4c^2 \iff a+b \geq 2c.$$

We now solve the problem:

**Claim.** We have $a+b \geq 2c$.

*Proof.* Because of distinctness, we may as well assume WLOG that

$$a > c > c-1 > b.$$

Since the function

$$\begin{aligned} f \colon (\sqrt{c(c-1)}, \infty) &\to \mathbb{R} \\ x &\mapsto x + \frac{c(c-1)}{x} \end{aligned}$$

is strictly increasing, it follows that

$$a+b = f(a) > f(c) = 2c-1 \implies a+b \geq 2c.$$ □