

Twitch 165.2

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TWITCH SOLVES ISL

Episode 165

Problem

Let f be a continuously differentiable function on $[0, \pi]$, such that $f(0) = f(\pi) = 0$.
Prove that

$$\int_0^\pi f'(x)^2 dx \geq \int_0^\pi f(x)^2 dx$$

with equality if and only if $f(x) = c \sin x$ for some constant c .

Solution

The idea is to integrate

$$\int_0^\pi (f'(x) - \cot x f(x))^2 dx \geq 0.$$

Details to be added.