# EGMO 2024/2


## Evan Chen


Twitch Solves ISL

Episode 146

## Problem

Let $ABC$ be a triangle with $AC > AB$, and denote its circumcircle by $\Omega$ and incenter by $I$. Let its incircle meet sides $BC$, $CA$, $AB$ at $D$, $E$, $F$ respectively. Let $X$ and $Y$ be two points on minor arcs $\widehat{DF}$ and $\widehat{DE}$ of the incircle, respectively, such that $\angle BXD = \angle DYC$. Let line $XY$ meet line $BC$ at $K$. Let $T$ be the point on $\Omega$ such that $KT$ is tangent to $\Omega$ and $T$ is on the same side of line $BC$ as $A$. Prove that lines $TD$ and $AI$ meet on $\Omega$.

## Video

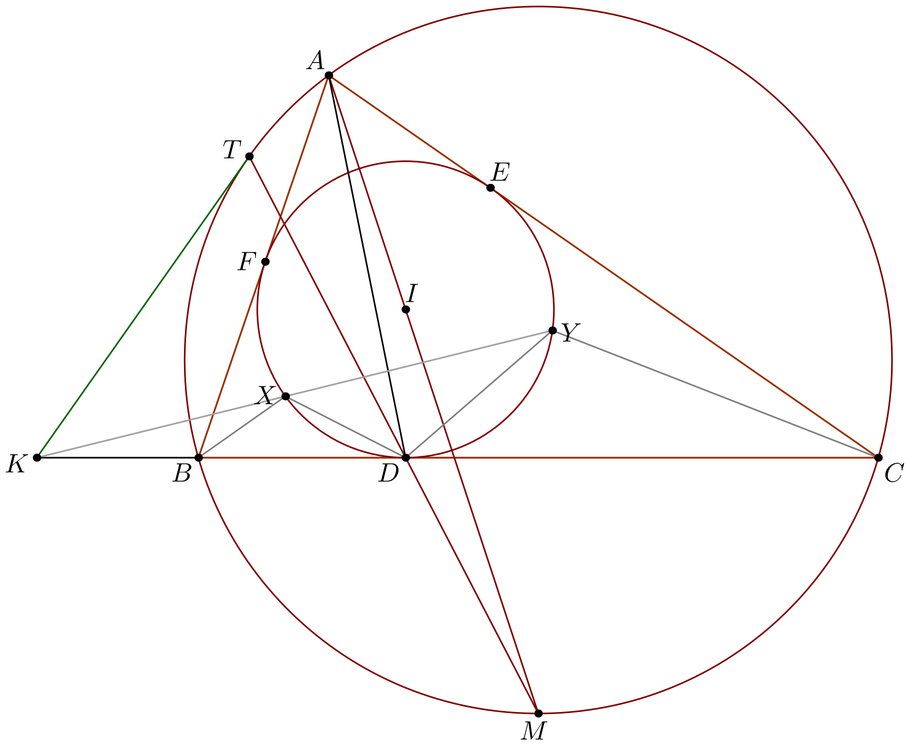
https://youtu.be/Q1CLri5l28I

## External Link

https://aops.com/community/p30433677

## Solution

We redefine the points in the reverse order: let $M$ be the arc midpoint of minor arc $BC$, let $T$ be the second intersection of ray $TD$ with $\Omega$, and let $K$ be the point on line $BC$ such that $\overline{KT}$ is tangent to $\Omega$. Then we will show that if a line through $K$ meets the incircle again at $X$ and $Y$, then $\angle BXD = \angle DYC$. A phantom point argument will then show the originas result.




**Claim.** $KT = KD$.

*Proof.* Note $\angle KTD = \frac{1}{2}\widehat{TM} = \frac{1}{2}(\widehat{TB} + \widehat{MC}) = \angle TDK$. □

Hence, it follows that

$$KX \cdot KY = KD^2 = KT^2 = KB \cdot KC$$

so quadrilateral $BXYC$ is cyclic. From this we deduce the inverse similarities

$$\triangle KXD \overset{-}{\sim} \triangle KDY$$
$$\triangle KXB \overset{-}{\sim} \triangle KCY.$$

Hence,

$$\angle BXD = \angle XKD - \angle KXB = \angle YDK - \angle YCK = \angle DYC$$

as needed.