

H3278100

Evan Chen

TWITCH SOLVES ISL

Episode 140

Problem

Let a, b, c be positive real numbers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$. Show that

$$a + b + c \leq \sqrt{b^3 + ac^2 + a^2c}.$$

Video

<https://youtu.be/sJsAqXuGj-A>

External Link

<https://aops.com/community/p30181272>

Solution

Note that by Cauchy-Schwarz the inequality

$$(b^3 + a^2c + ac^2) \left(\frac{1}{b} + \frac{1}{c} + \frac{1}{a} \right) \geq (b + a + c)^2$$

holds for all $a, b, c > 0$ which is what we wanted.