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Twitch Solves ISL

Episode 135

Problem

In triangle ABC, let I be the incenter, V be the Bevan point, and let the line through B and the de Longchamps point intersect AC again at X. Let D be the A-intouch point, and let ID intersect AB at P. Let BV intersect IX at Q, and let the reflection of ID over the midpoint of BC intersect AC at R. Prove that P, Q, and R are collinear.

Video

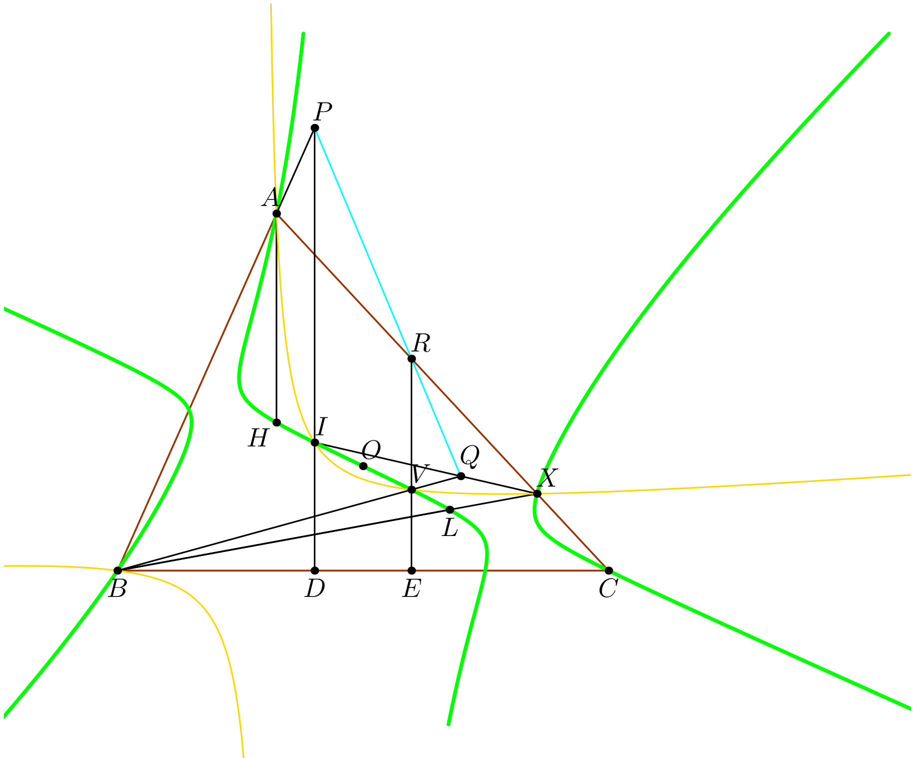
https://youtu.be/3tbbD4GaaHs

External Link

https://aops.com/community/p28829164

Solution

Introduce the Darboux cubic K_4. It's known to pass through the points A, B, the orthocenter H, the incenter I, the circumcenter O, the infinity point along the A-altitude (denoted ∞), the de Longchamps point L (which is the pivot), and the Bevan point V. Because L is the pivot, it also passes through X.



Consider now the following 3×3 array of points all lying on K_4:

$$\begin{bmatrix} A & \infty & H \\ I & V & O \\ X & B & L \end{bmatrix}$$

The three rows denote collinear points, as does the last column. So by Cayley-Bacharach the six points in the first two columns lie on a conic, say γ.

Then by Pascal theorem on

$$BV\infty IXA$$

we conclude P, Q, R are collinear, as desired.