

# H3167668

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TWITCH SOLVES ISL

Episode 135

## Problem

In triangle  $ABC$ , let  $I$  be the incenter,  $V$  be the Bevan point, and let the line through  $B$  and the de Longchamps point intersect  $AC$  again at  $X$ . Let  $D$  be the  $A$ -intouch point, and let  $ID$  intersect  $AB$  at  $P$ . Let  $BV$  intersect  $IX$  at  $Q$ , and let the reflection of  $ID$  over the midpoint of  $BC$  intersect  $AC$  at  $R$ . Prove that  $P$ ,  $Q$ , and  $R$  are collinear.

## Video

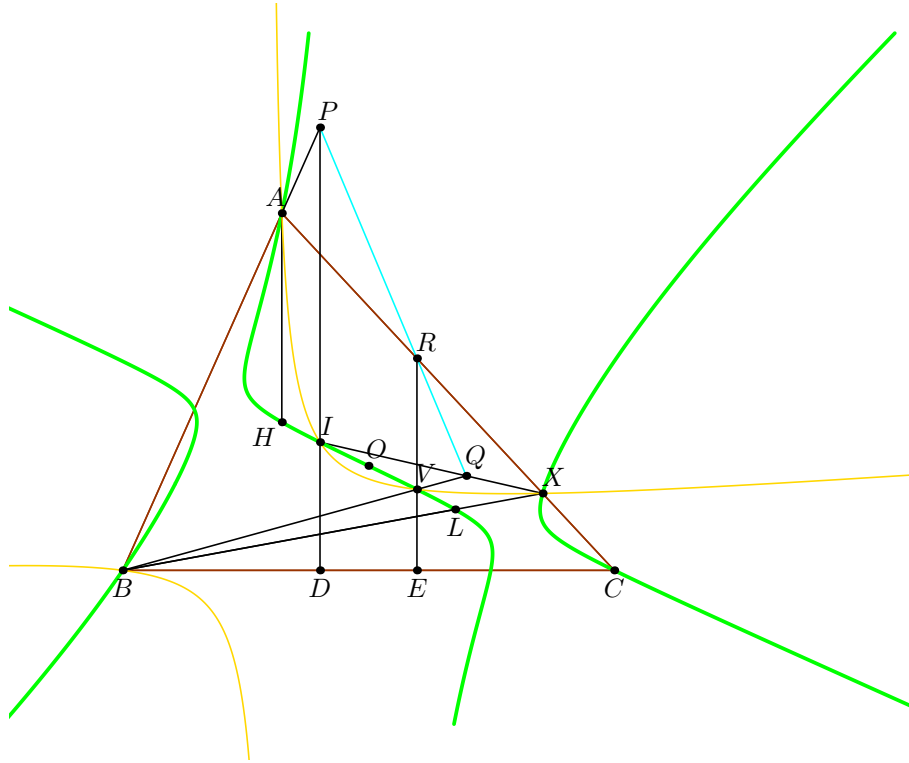
<https://youtu.be/3tbbD4GaaHs>

## External Link

<https://aops.com/community/p28829164>

### Solution

Introduce the Darboux cubic  $K_4$ . It's known to pass through the points  $A$ ,  $B$ , the orthocenter  $H$ , the incenter  $I$ , the circumcenter  $O$ , the infinity point along the  $A$ -altitude (denoted  $\infty$ ), the de Longchamps point  $L$  (which is the pivot), and the Bevan point  $V$ . Because  $L$  is the pivot, it also passes through  $X$ .



Consider now the following  $3 \times 3$  array of points all lying on  $K_4$ :

$$\begin{bmatrix} A & \infty & H \\ I & V & O \\ X & B & L \end{bmatrix}$$

The three rows denote collinear points, as does the last column. So by Cayley-Bacharach the six points in the first two columns lie on a conic, say  $\gamma$ .

Then by Pascal theorem on

$$BV\infty IXA$$

we conclude  $P$ ,  $Q$ ,  $R$  are collinear, as desired.