Twitch 129.3

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Twitch Solves ISL

Episode 129

Problem

Does there exist a set S of 4 circles, no three coaxial, such that there is exactly 4 circles tangent to all circles in S?

Video

https://youtu.be/hA8yUGtN0ks

Solution

Yes. Here is a construction.

Take a scalene triangle ABC, and its nine-point circle. Then there are exactly four circles tangent to line AB, BC, CA, and the nine-point circle, namely the incircle and its excircles.

Now invert around any point not on the lines AB, BC, CA, or the nine-point circle to transform this into four circles with the desired