Twitch 125.5

Evan Chen

Twitch Solves ISL

Episode 125

Problem

For which positive integers n is it possible to partition a square into n acute triangles?

Video

https://youtu.be/3D2kETEbaI3rM

External Link

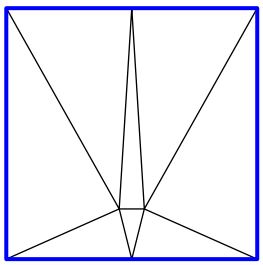
https://www.ics.uci.edu/~eppstein/junkyard/acute-square/

Solution

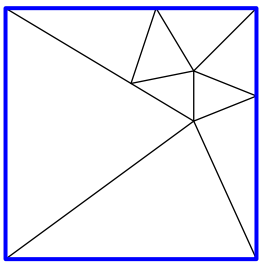
The answer is $n \geq 8$ only. The proof is divided into three parts.

Proof that $n \leq 7$ fails. I didn't write this up because it's too annoying. The idea is to classify a vertex as *bursting* if it lies strictly inside the square and has no 180° angle. This gives at least five triangles already.

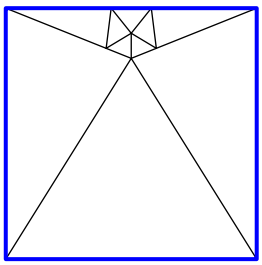
Construction for $n \in \{8, 9, 10\}$. See below.



$n = 8$



$n = 9$



$n = 10$

Inductive finish. Given a valid construction for n, one can take any triangle and add its medial triangle to get a valid construction for $n + 3$. This completes the proof.