

# Twitch 125.5

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TWITCH SOLVES ISL

Episode 125

## Problem

For which positive integers  $n$  is it possible to partition a square into  $n$  acute triangles?

## Video

<https://youtu.be/3D2kETEbaI3rM>

## External Link

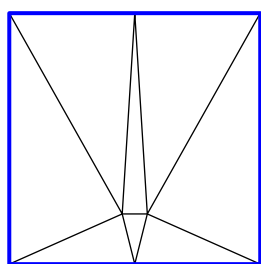
<https://www.ics.uci.edu/~epstein/junkyard/acute-square/>

## Solution

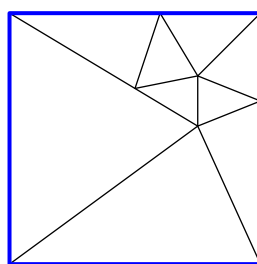
The answer is  $n \geq 8$  only. The proof is divided into three parts.

**Proof that  $n \leq 7$  fails.** I didn't write this up because it's too annoying. The idea is to classify a vertex as *bursting* if it lies strictly inside the square and has no  $180^\circ$  angle. This gives at least five triangles already.

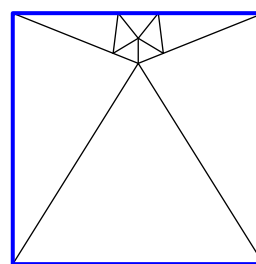
**Construction for  $n \in \{8, 9, 10\}$ .** See below.



$n = 8$



$n = 9$



$n = 10$

**Inductive finish.** Given a valid construction for  $n$ , one can take any triangle and add its medial triangle to get a valid construction for  $n + 3$ . This completes the proof.