

Iberoamerican 2022/2

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TWITCH SOLVES ISL

Episode 124

Problem

Let $S = \{13, 133, 1333, \dots\}$. Consider a horizontal row of 2022 cells. Ana and Borja play a game: they alternatively write a digit on the leftmost empty cell, starting with Ana. When the row is filled, the digits are read from left to right to obtain a 2022-digit number N . Borja wins if N is divisible by a number in S , otherwise Ana wins. Find which player has a winning strategy.

Video

<https://youtu.be/doMHBeuDnxQ>

External Link

<https://aops.com/community/p26230907>

Solution

Ana wins. All that's needed is:

Claim. On Ana's k th move for $k = 1, 2, \dots, 1011$, Ana can pick the digit to ensure the final number (no matter what happens after) is neither a multiple of

$$X = \underbrace{133\dots 33}_{2024-2k}$$

nor

$$Y = \underbrace{1333\dots 33}_{2025-2k}.$$

Proof. X eliminates at most 8 choices of digits while Y eliminates at most 1. \square