CodeForces 1667C

Evan Chen

Twitch Solves ISL

Episode 113

Problem

Let n be a positive integer. We wish to place m *half-queens* on an $n \times n$ chessboard; these can attack horizontally, vertically, or along the down-right/up-left diagonal (i.e. in six directions), and they attack the cells they occupy. Determine the smallest m needed so that there exists some placement in which every cell in the board is attacked by at least one half-queen.

Video

https://youtu.be/UEspJ8Xkmpo

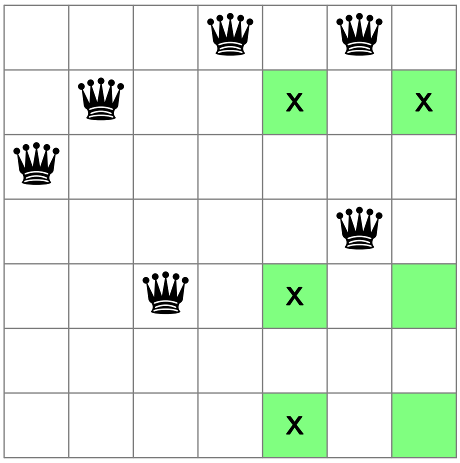
External Link

https://codeforces.com/problemset/problem/1667/C/

Solution

Answer: $\lceil \frac{2n-1}{3} \rceil$.

For the bound, color green any cell that is not in the same row or column as one of our m half-queens. That means we have (at least) an $(n-m) \times (n-m)$ array of green cells, and each green cell must be in a diagonal of a half-queen. In this array, observe that the leftmost column and topmost row of green cells cannot ever lie in the same diagonal (marked X below).

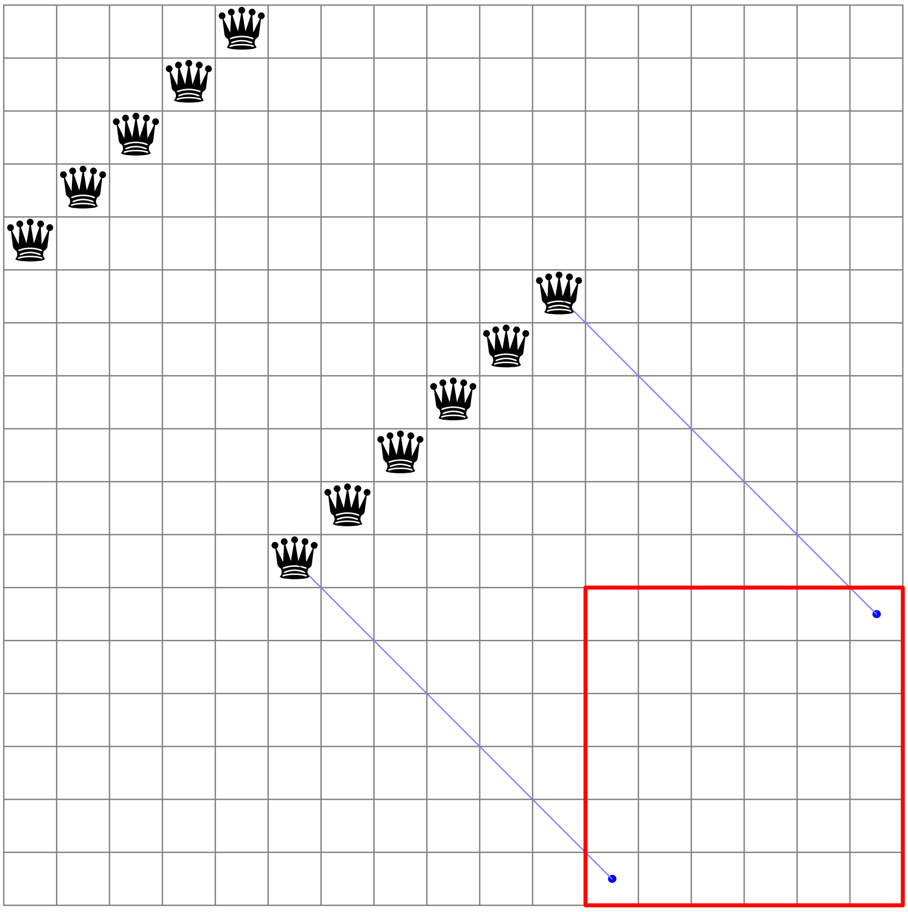


If the queens did cover all green cells, $2(n-m)-1$ half-queens. In other words,

$$m \geq 2(n-m) - 1 \implies m \geq \frac{2n-1}{3}$$

which implies the bound.

For the construction, consider first $n \equiv 2 \pmod 3$. We give a construction for $n = 17$ that generalizes ready.



When $n \equiv 1 \pmod 3$, use the construction for $(n+1) \times (n+1)$ and delete the rightmost row and bottom column. When $n \equiv 0 \pmod 3$, add a row and column at the top and left, and place a new queen in the upper-left hand corner.