CodeForces 1667C Evan Chen

TWITCH SOLVES ISL

Episode 113

Problem

Let n be a positive integer. We wish to place m half-queens on an $n \times n$ chessboard; these can attack horizontally, vertically, or along the down-right/up-left diagonal (i.e. in six directions), and they attack the cells they occupy. Determine the smallest m needed so that there exists some placement in which every cell in the board is attacked by at least one half-queen.

Video

https://youtu.be/UEspJ8Xkmpo

External Link

https://codeforces.com/problemset/problem/1667/C/

Solution

Answer: $\left\lceil \frac{2n-1}{3} \right\rceil$.

For the bound, color green any cell that is not in the same row or column as one of our m half-queens. That means we have (at least) an $(n - m) \times (n - m)$ array of green cells, and each green cell must be in a diagonal of a half-queen. In this array, observe that the leftmost column and topmost row of green cells cannot ever lie in the same diagonal (marked X below).

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	Ŵ			x		х
Ŵ						
					Ŵ	
		Ŵ		х		
				x		

If the queens did cover all green cells, 2(n-m) - 1 half-queens. In other words,

$$m \ge 2(n-m)-1 \implies m \ge \frac{2n-1}{3}$$

which implies the bound.

For the construction, consider first $n \equiv 2 \pmod{3}$. We give a construction for n = 17 that generalizes ready.



When $n \equiv 1 \pmod{3}$, use the construction for $(n+1) \times (n+1)$ and delete the rightmost row and bottom column. When $n \equiv 0 \pmod{3}$, add a row and column at the top and left, and place a new queen in the upper-left hand corner.