# CodeForces 1667C <br> Evan Chen 

Twitch Solves ISL

Episode 113

## Problem

Let $n$ be a positive integer. We wish to place $m$ half-queens on an $n \times n$ chessboard; these can attack horizontally, vertically, or along the down-right/up-left diagonal (i.e. in six directions), and they attack the cells they occupy. Determine the smallest $m$ needed so that there exists some placement in which every cell in the board is attacked by at least one half-queen.

## Video

https://youtu.be/UEspJ8Xkmpo

## External Link

https://codeforces.com/problemset/problem/1667/C/

## Solution

Answer: $\left\lceil\frac{2 n-1}{3}\right\rceil$.
For the bound, color green any cell that is not in the same row or column as one of our $m$ half-queens. That means we have (at least) an $(n-m) \times(n-m)$ array of green cells, and each green cell must be in a diagonal of a half-queen. In this array, observe that the leftmost column and topmost row of green cells cannot ever lie in the same diagonal (marked X below).


If the queens did cover all green cells, $2(n-m)-1$ half-queens. In other words,

$$
m \geq 2(n-m)-1 \Longrightarrow m \geq \frac{2 n-1}{3}
$$

which implies the bound.
For the construction, consider first $n \equiv 2(\bmod 3)$. We give a construction for $n=17$ that generalizes ready.


When $n \equiv 1(\bmod 3)$, use the construction for $(n+1) \times(n+1)$ and delete the rightmost row and bottom column. When $n \equiv 0(\bmod 3)$, add a row and column at the top and left, and place a new queen in the upper-left hand corner.

