CodeForces 1698F

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Twitch Solves ISL

Episode 110

Problem

There is a sequence $a = (a_1, \dots, a_n)$ of n real numbers. You may perform the following operation on it: choose two integers $1 \le \ell \le r \le n$ where $a_l = a_r$, and reverse the subsequence from the ℓ^{th} to the r^{th} element, i.e. change $(a_\ell, a_{\ell+1}, \dots, a_{r-1}, a_r)$ to $(a_r, a_{r-1}, \dots, a_{l+1}, a_\ell)$.

Given another sequence $b = (b_1, \dots, b_n)$, determine in $O(n \log n)$ time whether there exists finite sequence of moves changing a into b.

Video

https://youtu.be/h3CEfCfK924

External Link

https://codeforces.com/problemset/problem/1698/F

Solution

We give two invariants:

- The leftmost and rightmost elements obviously never change.
- Construct an undirected graph G on the set of real numbers by creating one edge $\{a_i, a_{i+1}\}$ for each $i = 1, \dots, n-1$, self-loops and multiple edges allowed (hence exactly $n-1$ edges). Then G never changes.

Claim. If these invariants are the same between a and b, then it's possible to take a to b.

Proof. By induction on n. Let $a_1 = b_1 = x$ and $b_2 = y$. We know x is next to y somewhere, so perform a move to make the second element of a equal to y. Then continue the induction. $\square$