# DEMO Mock 2022/1 

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## Twitch Solves ISL

Episode 107

## Problem

Determine all positive integers $N$ where the equations

$$
a b-c d=a+b+c+d=N
$$

have at least one solution for positive integers $a, b, c$, and $d$.

## Video

https://youtu.be/XUAVeDk2GsY

## External Link

https://aops.com/community/p25441644

## Solution

Ignore $N$ for a moment and write the equation as $(a-1)(b-1)=(c+1)(d+1)$. Then the factor lemma shows that it's equivalent to

$$
\underbrace{(a-1)}_{p q} \underbrace{(b-1)}_{r s}=\underbrace{(c+1)}_{p r} \underbrace{(d+1)}_{q s}
$$

So now $N=a+b+c+d=p q+r s+p r+q s=(p+s)(r+q)$.
The only caveat is we can't find $p=r=1$ or $q=s=1$. So we get that any composite $N$ other than $N=4$ or $N=6$ is OK.

