# Australia 2020/8 <br> Evan Chen 

## Twitch Solves ISL

Episode 107

## Problem

Prove that for each integer $k$ satisfying $2 \leq k \leq 100$, there are positive integers $b_{2}, b_{3}$, $\ldots, b_{101}$ such that

$$
b_{2}^{2}+b_{3}^{3}+\cdots+b_{k}^{k}=b_{k+1}^{k+1}+b_{k+2}^{k+2}+\cdots+b_{101}^{101}
$$

## Video

https://youtu.be/Qqk8cbL2fP8

## External Link

https://aops.com/community/p15841150

## Solution

For an integer $M$ to be chosen later, we will choose

$$
\begin{aligned}
b_{2}^{2} & =69696 M^{100!} \\
b_{3}^{3}=\cdots=b_{100}^{100} & =M^{100!} .
\end{aligned}
$$

(Note that $69696=264^{2}$.) Then the desired equation becomes

$$
b_{101}^{101}=(69696+(k-2)-(100-k)) \cdot M^{100!}
$$

and so we can let $M=69696+(k-2)-(100-k)$ and we're OK, since $M^{100!+1}$ is obviously a 100th power by Wilson's theorem.

