# Twitch 106.4

**Evan Chen**

Twitch Solves ISL

Episode 106

## Problem

Given an equilateral triangle $ABC$ and a point $P$ within ABC, construct the perpendicular $\ell_A$ to $AP$ through $P$. Let $A_1$ and $A_2$ be the intersections of $\ell_A$ through $AB$ and $AC$, respectively, and let the intersection of $BA_2$ and $CA_1$ be called $A'$. Construct $B'$ and $C'$ in a similar manner. Prove that $AA'$, $BB'$, and $CC'$ are concurrent.

## Video

https://youtu.be/_HtErqKU_zQ

## Solution

Let line $AA'$ meet line $BC$ at $X$; also let $\ell_A \cap BC = X'$.

**Claim 1.** $(BC;XX') = -1$.

*Proof.* Harmonic bundle picture. $\square$

So by Ceva's theorem, to prove the concurrence, it's equivalent (by Menelaus) to prove the collinearity of $X'Y'Z'$.

But this line has a name: it's the *orthotransversal* of $P$ with respect to $\triangle ABC$.