# Twitch 106.4 

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Twitch Solves ISL
Episode 106

## Problem

Given an equilateral triangle $A B C$ and a point $P$ within ABC, construct the perpendicular $\ell_{A}$ to $A P$ through $P$. Let $A_{1}$ and $A_{2}$ be the intersections of $\ell_{A}$ through $A B$ and $A C$, respectively, and let the intersection of $B A_{2}$ and $C A_{1}$ be called $A^{\prime}$. Construct $B^{\prime}$ and $C^{\prime}$ in a similar manner. Prove that $A A^{\prime}, B B^{\prime}$, and $C C^{\prime}$ are concurrent.

## Video

https://youtu.be/_HtErqKU_zQ

## Solution

Let line $A A^{\prime}$ meet line $B C$ at $X$; also let $\ell_{A} \cap B C=X^{\prime}$.
Claim 1. $\left(B C ; X X^{\prime}\right)=-1$.
Proof. Harmonic bundle picture.
So by Ceva's theorem, to prove the concurrence, it's equivalent (by Menelaus) to prove the collinearity of $X^{\prime} Y^{\prime} Z^{\prime}$.

But this line has a name: it's the orthotransversal of $P$ with respect to $\triangle A B C$.

