## Twitch 106.4 Evan Chen

TWITCH SOLVES ISL

Episode 106

## Problem

Given an equilateral triangle ABC and a point P within ABC, construct the perpendicular  $\ell_A$  to AP through P. Let  $A_1$  and  $A_2$  be the intersections of  $\ell_A$  through AB and AC, respectively, and let the intersection of  $BA_2$  and  $CA_1$  be called A'. Construct B' and C' in a similar manner. Prove that AA', BB', and CC' are concurrent.

## Video

https://youtu.be/\_HtErqKU\_zQ

## Solution

Let line AA' meet line BC at X; also let  $\ell_A \cap BC = X'$ .

Claim 1. (BC; XX') = -1.

*Proof.* Harmonic bundle picture.

So by Ceva's theorem, to prove the concurrence, it's equivalent (by Menelaus) to prove the collinearity of X'Y'Z'.

But this line has a name: it's the *orthotransversal* of P with respect to  $\triangle ABC$ .