# Brazil 2022/2 <br> Evan Chen <br> Twitch Solves ISL <br> Episode 106 

## Problem

Let $A B C$ be an acute triangle, with $A B<A C$. Let $K$ be the midpoint of the arc $B C$ that does not contain $A$ and let $P$ be the midpoint of $B C$. Let $I_{B}, I_{C}$ be the $B$-excenter and $C$-excenter of $A B C$, respectively. Let $Q$ be the reflection of $K$ with respect to $A$. Prove that the points $P, Q, I_{B}, I_{C}$ are concyclic.

## Video

https://youtu.be/w0Hvkqf0aQ8

## External Link

https://aops.com/community/c6h2965497p26563619

## Solution

Let $I_{A}$ be the $A$-excenter, and let $H=\overline{A I K} \cap \overline{B C}$.


- Then by Brokard's theorem on cyclic quadrilateral $B I C I_{A}$, it follows that $\triangle I_{B} I_{C} H$ is self-polar to this circle.
- In particular $K$ is the orthocenter. Equivalently, $H$ is the orthocenter of $\triangle I_{B} K I_{C}$.
- Define $T=\overline{I_{B} I_{C}} \cap \overline{B C}$. Then $(T H ; B C)=-1$.
- I claim that $I_{B} I_{C} P H$ is cyclic. Indeed, $T H \cdot T P=T B \cdot T C=T I_{B} \cdot I_{C}$.
- The reflection of the orthocenter $K$ of $\Delta I_{B} I_{C} H$ over a side $\overline{I_{B} I_{C}}$ then coincides with $Q$, which now lies on $\left(P H I_{B} I_{C}\right)$.

