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Twitch Solves ISL

Episode 104

Problem

Let ABC be an acute triangle with circumcircle Γ. Let P and Q be points in the half plane defined by BC containing A, such that BP and CQ are tangents to Γ and $PB = BC = CQ$. Let K and L be points on the external bisector of $\angle CAB$, such that $BK = BA$, $CL = CA$. Let M be the intersection point of the lines PK and QL. Prove that $MK = ML$.

Video

https://youtu.be/dhCir0TYqIg

External Link

https://aops.com/community/p26232444

Solution

Let Y be the arc midpoint of arc BAC.

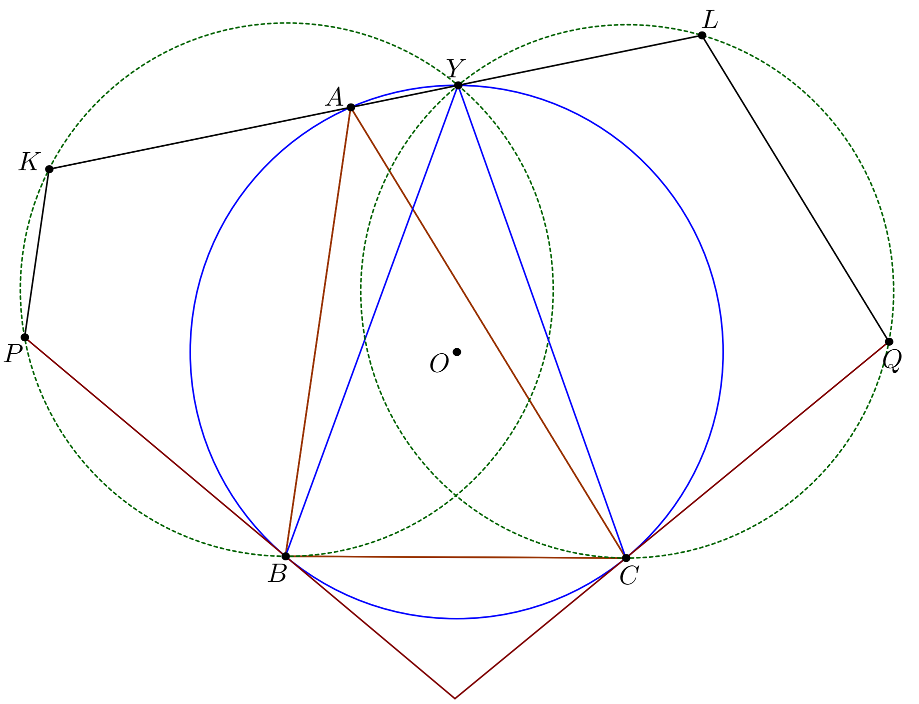
Claim. P is reflection of C across line YB.

Proof. It's easy to check $\overline{BY}$ bisects $\angle PBC$. $\square$

Claim. $PKBY$ is cyclic.

Proof. $\measuredangle BPY = \measuredangle YCB = \measuredangle YAB = \measuredangle KAB = \measuredangle BKA = \measuredangle BKY$. $\square$

Now $\measuredangle PKY = \measuredangle PBY = \measuredangle YCQ = \measuredangle YLQ$ so $\triangle MKL$ has isosceles base angles.



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Remark. Conjectures to prove (APQ) is tangent to $\overline{KL}$ and M lies on the A-symmedian.