IMO 1997/2

Evan Chen

Twitch Solves ISL

Episode 101

Problem

It is known that $\angle BAC$ is the smallest angle in the triangle ABC. The points B and C divide the circumcircle of the triangle into two arcs. Let U be an interior point of the arc between B and C which does not contain A. The perpendicular bisectors of AB and AC meet the line AU at V and W, respectively. The lines BV and CW meet at T.

Show that $AU = TB + TC$.

Video

https://youtu.be/HytKqrMVGpc

Solution

Let $\overline{BTV}$ meet the circle again at U_1, so that AU_1UB is an isosceles trapezoid. Define U_2 similarly.

Now from the isosceles trapezoids we get

$$AU = BU_1 = BT + TU_1 = BT + TC$$

as desired.