

# IMO 1997/2

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TWITCH SOLVES ISL

Episode 101

## Problem

It is known that  $\angle BAC$  is the smallest angle in the triangle  $ABC$ . The points  $B$  and  $C$  divide the circumcircle of the triangle into two arcs. Let  $U$  be an interior point of the arc between  $B$  and  $C$  which does not contain  $A$ . The perpendicular bisectors of  $AB$  and  $AC$  meet the line  $AU$  at  $V$  and  $W$ , respectively. The lines  $BV$  and  $CW$  meet at  $T$ .

Show that  $AU = TB + TC$ .

## Video

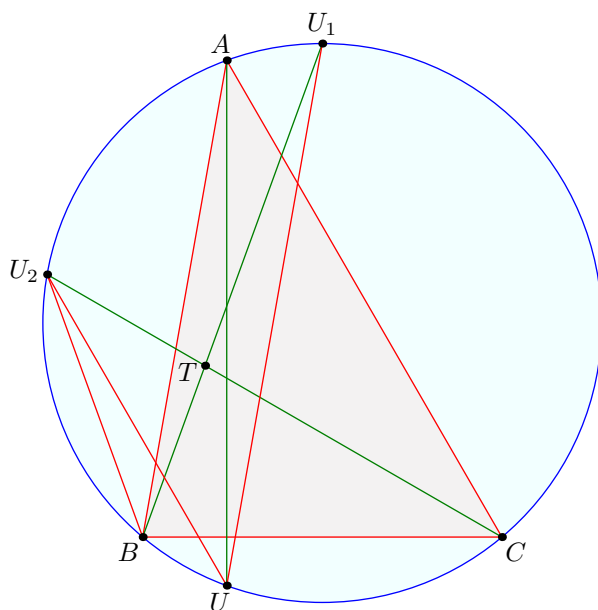
<https://youtu.be/HytKqrMVGpc>

## External Link

<https://aops.com/community/p356701>

## Solution

Let  $\overline{BTU}$  meet the circle again at  $U_1$ , so that  $AU_1UB$  is an isosceles trapezoid. Define  $U_2$  similarly.



Now from the isosceles trapezoids we get

$$AU = BU_1 = BT + TU_1 = BT + TC$$

as desired.