ToT Fall 2005 S-A3

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TWITCH SOLVES ISL

Episode 74

Problem

Originally, every square of 8×8 chessboard contains a rook. One by one, rooks which attack an odd number of others are removed. Find the maximal number of rooks that can be removed. (A rook attacks another rook if they are on the same row or column and there are no other rooks between them.)

Video

https://youtu.be/vH67pRJdM8g

Solution

The answer is 59.

Proof ≥ 5 rooks always remain. To show that 5 rooks must remain, observe first that the four corners may never be deleted (as long as all four corners are present, these rooks attack exactly 2 others). Moreover, if there are 5 rooks left on the board, the non-corner rook cannot be removed by inspection. $\square$

Here is the construction. Each cell is labeled with the time it is removed. They are color coded for convenience.

$$\begin{bmatrix} \bigstar & 1 & 2 & 3 & 4 & 5 & 6 & \bigstar \\ 13 & 50 & 51 & 52 & 53 & 54 & 55 & 7 \\ 14 & 15 & 16 & 17 & 18 & 19 & 56 & 8 \\ 20 & 21 & 22 & 23 & 24 & 25 & 57 & 9 \\ 26 & 27 & 28 & 29 & 30 & 31 & 58 & 10 \\ 32 & 33 & 34 & 35 & 36 & 37 & 59 & 11 \\ 38 & 39 & 40 & 41 & 42 & 43 & \bigstar & 12 \\ \bigstar & 44 & 45 & 46 & 47 & 48 & 49 & \bigstar \end{bmatrix}$$