# Russia 2010/10.8 <br> Evan Chen 

## Twitch Solves ISL

Episode 73

## Problem

Let $G$ be a finite connected graph. Suppose that, if we choose any odd cycle in $G$ and delete all its edges, the resulting graph is not connected. Prove that we color each vertex of $G$ with one of four colors such that no two adjacent vertices are the same color.

## Video

https://youtu.be/uDVCCdQLt6s

## External Link

https://aops.com/community/p2013543

## Solution

Let $T$ be a spanning tree, and let $H$ be the subgraph (not induced) of remaining edges.

- Trees are bipartite, so $T$ has a two-coloring.
- By condition, $H$ has no odd cycles (since deletion of an odd cycle should disconnect $G)$. Hence $H$ is bipartite, and has a two-coloring.

Then we can four-color $G$ using the ordered pair of two-colorings.

