

# Sweden 2018

Evan Chen

TWITCH SOLVES ISL

Episode 72

## Problem

For which positive integers  $n$  is the polynomial

$$p(x) = 1 + x^n + x^{2n}$$

reducible over the integers?

## Video

<https://youtu.be/drrHDLWR6bw>

## Solution

The answer is only  $n$  a power of 3 (including 1). Indeed, letting  $\Phi_\bullet$  denote the cyclotomic polynomial, we have the decomposition of  $p$  into irreducibles is exactly given by

$$p(x) = \frac{x^{3n} - 1}{x^n - 1} = \frac{\prod_{d|3n} \Phi_d(n)}{\prod_{d|n} \Phi_d(n)} = \prod_{\substack{d|3n \\ d \nmid n}} \Phi_d(n).$$

Hence the answer is those  $n$  for which there is only one  $d$  satisfying  $d \mid 3n$ ,  $d \nmid n$ . This is exactly the powers of 3.