

# JBMO SL 2008 A2

Evan Chen

TWITCH SOLVES ISL

Episode 71

## Problem

Find all real numbers  $a, b, c, d$  such that

$$a + b + c + d = 20$$

$$ab + bc + cd + da + ac + bd = 150.$$

## Video

<https://youtu.be/dlrSmf05n-w>

**Solution**

Note that

$$a^2 + b^2 + c^2 + d^2 = 20^2 - 2 \cdot 150 = 100$$

but by Cauchy-Schwarz

$$100 \cdot 4 = (a^2 + b^2 + c^2 + d^2)(1 + 1 + 1 + 1) \geq (a + b + c + d)^2 = 400$$

so equality must hold meaning  $a = b = c = d = 5$ , and this clearly works.