# Sharygin 2019/23

**Evan Chen**

Twitch Solves ISL

Episode 62

## Problem

In the plane, let $a$, $b$ be two closed broken lines (possibly self-intersecting), and $K$, $L$, $M$, $N$ be four points. The vertices of $a$, $b$ and the points $K$, $L$, $M$, $N$ are in general position (i.e. no three of these points are collinear, and no three segments between them concur at an interior point). Each of segments $KL$ and $MN$ meets $a$ at an even number of points, and each of segments $LM$ and $NK$ meets $a$ at an odd number of points. Conversely, each of segments $KL$ and $MN$ meets $b$ at an odd number of points, and each of segments $LM$ and $NK$ meets $b$ at an even number of points. Prove that $a$ and $b$ intersect.

## Video

https://youtu.be/oCtUUKGXuaA

## Solution

Assume for contradiction this is not so.

**Claim** (Well-known)**.** The curve $a$ encloses a region (meaning one can discuss being inside or outside $a$), and similarly for $b$.

Now:

- Since $KN$ intersects $a$ an odd number of times, exactly one of the two points is inside $a$. WLOG $K$ is inside $a$ and $N$ is outside.
- Following through, $M$ is outside, so $L$ is inside.
- But then $KL$ can't intersect $b$ at all, contradiction.