# Twitch 059.1

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Twitch Solves ISL

Episode 59

## Problem

Find all functions $f\colon \mathbb{Z}_{\geq 0} \to \mathbb{R}$ such that

$$f(2x+3y) = 3f(x) + 2f(y)$$

holds.

## Video

https://youtu.be/wqNNrhbVlvc

## Solution

The only solution is $f \equiv 0$, which works.

By plugging in zeros, we get $f(0) = 0$, $f(2t) = 3f(t)$, $f(3t) = 2f(t)$. Also, $x = y = t$ gives $f(5t) = 5f(t)$.

Now, for any $t \in \mathbb{R}$, let $y = f(t)$. Then

$$\begin{aligned} f(10t + 3t) &= f(4t + 9t) \\ 3f(5t) + 2f(t) &= 3f(2t) + 2f(3t) \\ 15y + 2y &= 9y + 4y \end{aligned}$$

Thus $f \equiv 0$.