# Brazil 2013/2 

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## Problem

Arnaldo and Bernaldo play the following game: given a fixed finite set of positive integers $A$ known by both players, Arnaldo picks a number $a \in A$ but doesn't tell it to anyone. Bernaldo then picks an arbitrary positive integer $b$ (not necessarily in $A$ ). Then Arnaldo tells the number of divisors of $a b$. Show that Bernaldo can choose $b$ in a way that he can find out the number $a$ chosen by Arnaldo.

## Video

https://youtu.be/TqUF1bW8cgE

## External Link

https://aops.com/community/p3256067

## Solution

Let $p_{0}, p_{1}, p_{2}, \ldots, p_{k}$ be the set of primes that appear anywhere in $A$. Define a huge integer $N$, such that

$$
N>\prod_{i} \max _{a \in A} \nu_{p_{i}}(a)
$$

for example (but anything big enough will do).
Then, we have Bernaldo select

$$
b=p_{0}^{N-1} p_{1}^{N^{2}-1} p_{2}^{N^{4}-1} p_{3}^{N^{8}-1} \ldots p_{k}^{N^{2^{k}}-1} .
$$

Then, if Arnaldo had picked $a=p_{0}^{e_{0}} \ldots p_{k}^{e_{k}}$, we would have

$$
d(a b)=\left(N+e_{0}\right)\left(N^{2}+e_{1}\right)\left(N^{4}+e_{2}\right)\left(N^{8}+e_{3}\right) \ldots\left(N^{2^{k}}+e_{k}\right) .
$$

If we view this number in base $N$, then there is no re-grouping, because $N$ is so large.
Then the $N^{2^{k+1}-1-2^{i}}$ digit would give $e_{i}$, so Bernaldo wins.
Remark. In general, Bernaldo can read any product from the digits: for example the $N^{5}$ 's place would give $e_{1} e_{3} e_{4} e_{5} \ldots e_{k}$, or the $N^{13}$ place would give $e_{1} e_{4} e_{5} \ldots e_{k}$. So only a few digits are necessary.

