Iberoamerican 2020/6

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TWITCH SOLVES ISL

Episode 51

Problem

Let ABC be an acute, scalene triangle. Let H be the orthocenter and O be the circumcenter of triangle ABC, and let P be a point interior to the segment HO. The circle with center P and radius PA intersects the lines AB and AC again at R and S, respectively. Denote by Q the symmetric point of P with respect to the perpendicular bisector of BC. Prove that points P, Q, R and S lie on the same circle.

External Link

https://aops.com/community/p18969935

Solution

By IMO Shortlist 2016 G5, one finds that the circumcenter of $\triangle PSR$ lies on the perpendicular bisector of \overline{BC} , the end.