

Iberoamerican 2020/6

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TWITCH SOLVES ISL

Episode 51

Problem

Let ABC be an acute, scalene triangle. Let H be the orthocenter and O be the circumcenter of triangle ABC , and let P be a point interior to the segment HO . The circle with center P and radius PA intersects the lines AB and AC again at R and S , respectively. Denote by Q the symmetric point of P with respect to the perpendicular bisector of BC . Prove that points P , Q , R and S lie on the same circle.

External Link

<https://aops.com/community/p18969935>

Solution

Let P' , R' , S' be the midpoints of AP , AR , AS . Apply **IMO Shortlist 2016 G5** to the circle centered at P' through R' and S' to deduce that the circumcenter of $\triangle P'S'R'$ lies halfway between the A -altitude and the perpendicular bisector of \overline{BC} .

Thus the circumcenter of $\triangle PSR$ lies on the perpendicular bisector of \overline{BC} . So it passes through Q .