## CMIMC 2017 G9 Evan Chen

TWITCH SOLVES ISL

Episode 48

## Problem

Let  $\triangle ABC$  be an acute triangle with circumcenter O, and let  $Q \neq A$  denote the point on  $\bigcirc(ABC)$  for which  $AQ \perp BC$ . The circumcircle of  $\triangle BOC$  intersects lines AC and AB for the second time at D and E respectively. Suppose that AQ, BC, and DE are concurrent. If OD = 3 and OE = 7, compute AQ.

## Video

https://youtu.be/-beq\_Y\_Npsw

## Solution

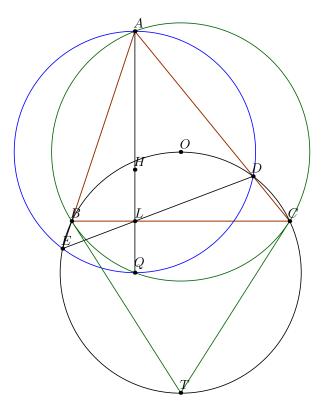
Let L be the concurrency point, so by radical axis AEQD is cyclic.

**Claim.** Line DLE is the Simson line from Q to ABC.

Proof. Note

$$\measuredangle BED = \measuredangle BCD = \measuredangle BCA = \measuredangle BQA = \measuredangle BQL$$

so BEQL is cyclic, hence  $\measuredangle QEB = \measuredangle QLB = 90^{\circ}$ , as desired.



Claim. *OEQD* is a parallelogram.

*Proof.* Since  $\angle OED = \angle OCD = 90^\circ - A$ , we find  $\overline{OE} \perp \overline{AC}$ . But  $\overline{QD} \perp \overline{AC}$  too.  $\Box$ 

The parallelogram law now gives

$$DE^2 + OQ^2 = 2(OD^2 + OE^2).$$

Also since O is the orthocenter of  $\triangle ADE$  it is known that

$$AO^2 = AQ^2 - DE^2.$$

Putting these two together and noting AO = OQ gives  $AQ^2 = 2(OD^2 + OE^2) = 116$ , hence  $AQ = 2\sqrt{29}$ .