# CMIMC 2017 G9 

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## Twitch Solves ISL

Episode 48

## Problem

Let $\triangle A B C$ be an acute triangle with circumcenter $O$, and let $Q \neq A$ denote the point on $\odot(A B C)$ for which $A Q \perp B C$. The circumcircle of $\triangle B O C$ intersects lines $A C$ and $A B$ for the second time at $D$ and $E$ respectively. Suppose that $A Q, B C$, and $D E$ are concurrent. If $O D=3$ and $O E=7$, compute $A Q$.

## Video

https://youtu.be/-beq_Y_Npsw

## Solution

Let $L$ be the concurrency point, so by radical axis $A E Q D$ is cyclic.
Claim. Line $D L E$ is the Simson line from $Q$ to $A B C$.
Proof. Note

$$
\measuredangle B E D=\measuredangle B C D=\measuredangle B C A=\measuredangle B Q A=\measuredangle B Q L
$$

so $B E Q L$ is cyclic, hence $\measuredangle Q E B=\measuredangle Q L B=90^{\circ}$, as desired.


Claim. $O E Q D$ is a parallelogram.
Proof. Since $\measuredangle O E D=\measuredangle O C D=90^{\circ}-A$, we find $\overline{O E} \perp \overline{A C}$. But $\overline{Q D} \perp \overline{A C}$ too.
The parallelogram law now gives

$$
D E^{2}+O Q^{2}=2\left(O D^{2}+O E^{2}\right) .
$$

Also since $O$ is the orthocenter of $\triangle A D E$ it is known that

$$
A O^{2}=A Q^{2}-D E^{2}
$$

Putting these two together and noting $A O=O Q$ gives $A Q^{2}=2\left(O D^{2}+O E^{2}\right)=116$, hence $A Q=2 \sqrt{29}$.

