# PUMaC 2015 C8 

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## Problem

In a tournament with 2015 teams, each team plays every other team exactly once and no ties occur. Such a tournament is imbalanced if for every group of 6 teams, there exists either a team that wins against the other 5 or a team that loses to the other 5 . If the teams are indistinguishable, what is the number of distinct imbalanced tournaments that can occur?

## Video

https://youtu.be/YuGZ _XHtL8

## Solution

We need the following lemma:
Lemma (Twitch Lemma). In a tournament if there is a $(k+1)$-cycle then there is a $k$-cycle.

Claim. A tournament is balanced if there exists two disjoint directed cycles, or a directed cycle of length at least 6 .

Proof. In the former case, the Twitch Lemma gives us two directed triangles, which is bad.

In the latter case, the Twitch Lemma gives us a directed 6 -cycle, which is bad.
Using this one can show (many details omitted) that the imbalanced tournaments are given by the following list:

- One fully ordered tournament
- $2013 \cdot 1$ tournaments that consist of a directed 3 -cycle and everything else being fully ordered.
- $2012 \cdot 1$ tournaments that consist of a directed 4 -cycle and everything else being fully ordered.
- $2011 \cdot 6$ tournament that consist of a directed 5 -cycle and everything else being fully ordered.

This gives an answer of $1+2013+2012+2011 \cdot 6=16092$.

