# USAMO 1999/1 

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## Twitch Solves ISL

Episode 41

## Problem

Some checkers placed on an $n \times n$ checkerboard satisfy the following conditions:
(a) every square that does not contain a checker shares a side with one that does;
(b) given any pair of squares that contain checkers, there is a sequence of squares containing checkers, starting and ending with the given squares, such that every two consecutive squares of the sequence share a side.

Prove that at least $\left(n^{2}-2\right) / 3$ checkers have been placed on the board.

## Video

https://youtu.be/Uv4AtkRVPwo

## External Link

https://aops.com/community/p340035

## Solution

Take a spanning tree on the set $V$ of checkers where the $|V|-1$ edges of the tree are given by orthogonal adjacency. By condition (a) we have

$$
\sum_{v \in V}(4-\operatorname{deg} v) \geq n^{2}-|V|
$$

and since $\sum_{v \in V} \operatorname{deg} v=2(|V|-1)$ we get

$$
4|V|-(2|V|-2) \geq n^{2}-|V|
$$

which implies $|V| \geq \frac{n^{2}-2}{3}$.

