

Twitch 032.3

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TWITCH SOLVES ISL

Episode 32

Problem

Square $DEAL$, with side length 2, is drawn, sharing a vertex with unit square $DIPS$. $DIPS$ has centroid R . Finally, square $RENT$ is drawn. If all of these squares were drawn in counterclockwise order of vertices, and IAN is a line, find the possible values for the side length of $RENT$.

Video

https://youtu.be/Nrj2bc9L_Tw

Solution

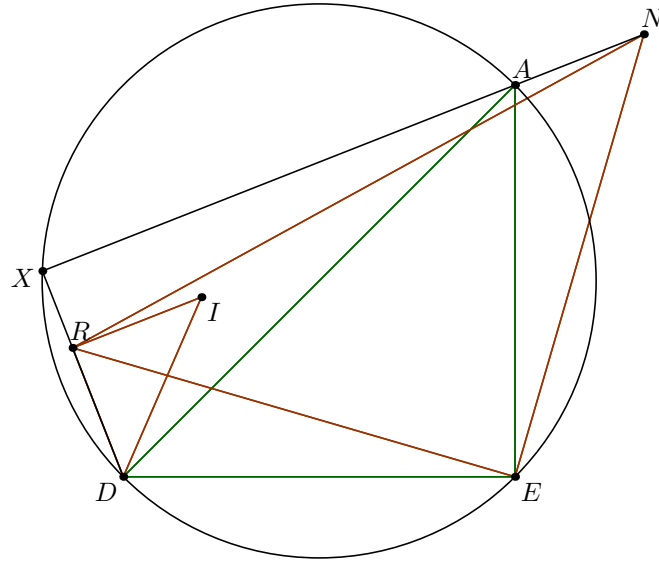
We delete the squares, basically, and focus on $45^\circ - 45^\circ - 90^\circ$ triangles DRI , DEA REN . Let us define X as the intersection of \overline{DR} and (DAE) .

The spiral similarity between DEA and REN gives a rotation

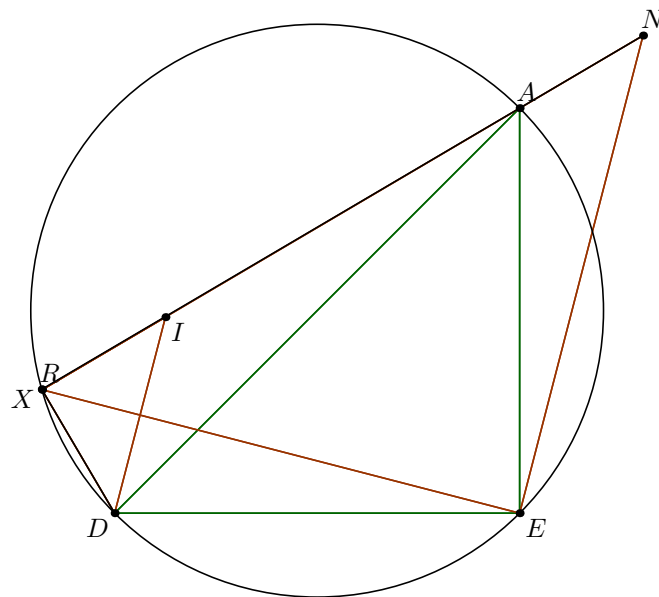
$$\triangle RDE \cong \triangle NAE$$

with $RD = NA = 1/\sqrt{2}$ and $DE = EN = 2$.

By the displayed rotation we have $\overline{NA} \perp \overline{RD}$, so if we let X be their intersection, we get a right angle. Hence IR and XAN are parallel.



Hence if I lies on line AN , we need to actually have $X = R$. One of two possibilities is shown below (in the other, X is on the other side of line AD).



The extraction is Ptolemy's theorem, left as exercise. Answer is $\frac{1}{2}(\sqrt{15} \pm 1)$.