BAMO 2003/5

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Twitch Solves ISL

Episode 31

Problem

Let $ABCD$ be a square, and let E be an internal point on side AD. Let F be the foot of the perpendicular from B to CE. Suppose G is a point such that $BG = FG$, and the line through G parallel to BC passes through the midpoint of EF. Prove that $AC < 2 \cdot FG$.

Video

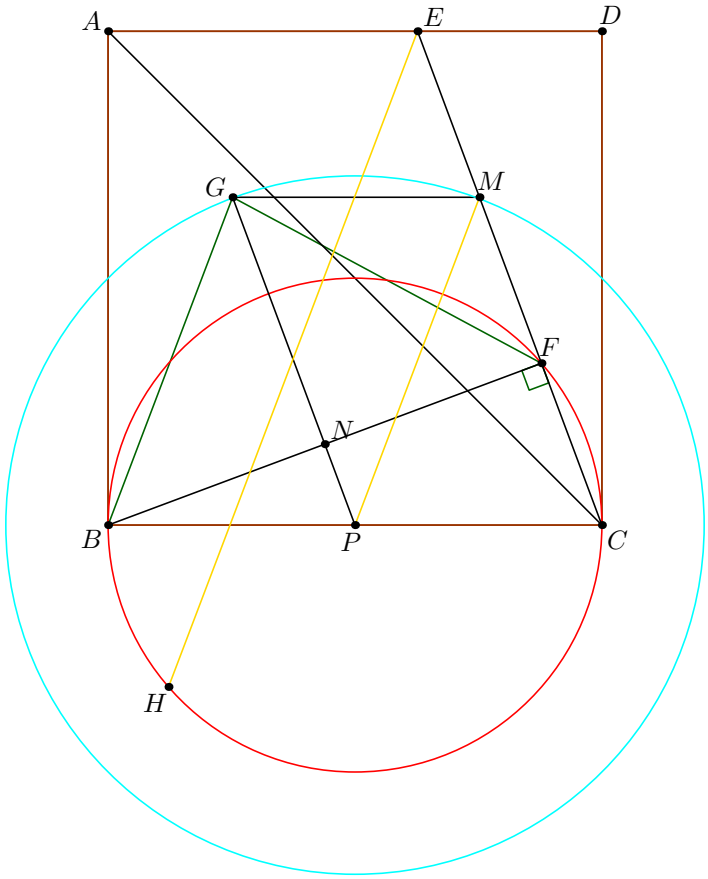
https://youtu.be/jZLZjmSOmRg

Solution

Since the opposite sides are parallel, it's clear that $GMCP$ is a parallelogram. Let H be the antipode of F. Then

$$FG = BG = MP = \frac{1}{2}EH$$

where H is the antipode of F.



So now,

$$EH^2 = EC^2 + CH^2 = BF^2 + CE^2 \overset{?}{>} BC^2 + CD^2$$

which is true because $\triangle BFC \sim \triangle CED$ and $BC < EC$.