

# BAMO 2003/5

Evan Chen

TWITCH SOLVES ISL

Episode 31

## Problem

Let  $ABCD$  be a square, and let  $E$  be an internal point on side  $AD$ . Let  $F$  be the foot of the perpendicular from  $B$  to  $CE$ . Suppose  $G$  is a point such that  $BG = FG$ , and the line through  $G$  parallel to  $BC$  passes through the midpoint of  $EF$ . Prove that  $AC < 2FG$ .

## Video

<https://youtu.be/jZLZjmS0mRg>

## External Link

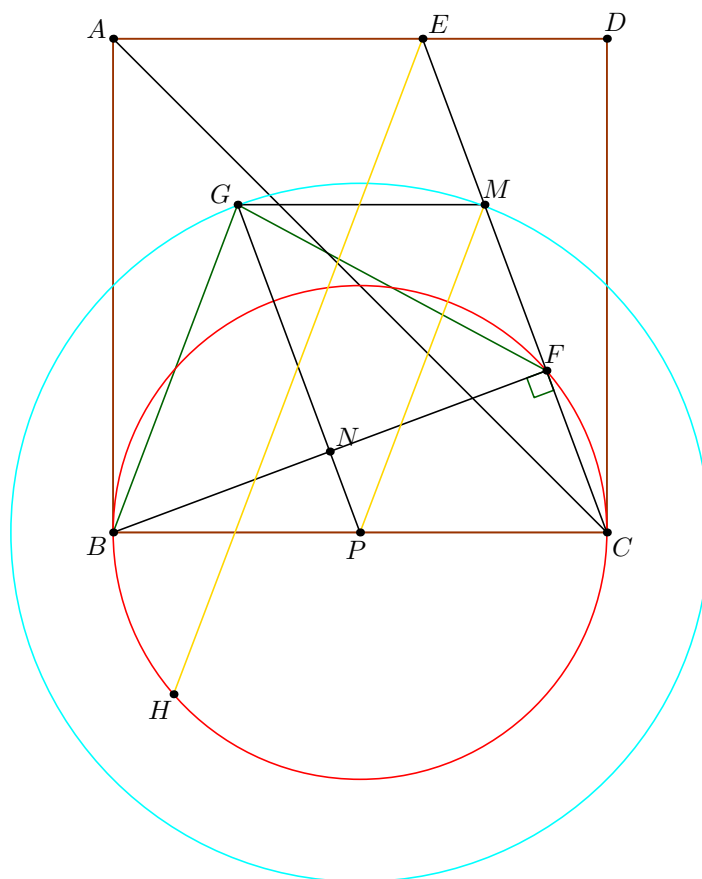
<https://aops.com/community/p13027339>

### Solution

Since the opposite sides are parallel, it's clear that  $GMCP$  is a parallelogram. Let  $H$  be the antipode of  $F$ . Then

$$FG = BG = MP = \frac{1}{2}EH$$

where  $H$  is the antipode of  $F$ .



So now,

$$EH^2 = EC^2 + CH^2 = BF^2 + CE^2 \stackrel{?}{>} BC^2 + CD^2$$

which is true because  $\triangle BFC \sim \triangle CED$  and  $BC < EC$ .