

# IMO 2020/1

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TWITCH SOLVES ISL

Episode 30

## Problem

Consider the convex quadrilateral  $ABCD$ . The point  $P$  is in the interior of  $ABCD$ . The following ratio equalities hold:

$$\angle PAD : \angle PBA : \angle DPA = 1 : 2 : 3 = \angle CBP : \angle BAP : \angle BPC.$$

Prove that the following three lines meet in a point: the internal bisectors of angles  $\angle ADP$  and  $\angle PCB$  and the perpendicular bisector of segment  $AB$ .

## Video

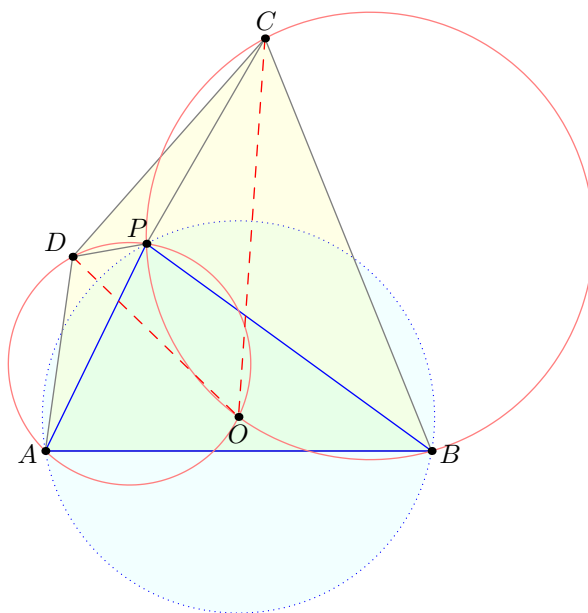
<https://youtu.be/JfRrlvbzKHk>

## External Link

<https://aops.com/community/p17821635>

## Solution

Let  $O$  denote the circumcenter of  $\triangle PAB$ . We claim it is the desired concurrency point.



Indeed,  $O$  obviously lies on the perpendicular bisector of  $AB$ . Now

$$\begin{aligned}\angle BCP &= \angle CBP + \angle BPC \\ &= 2\angle BAP = \angle BOP\end{aligned}$$

it follows  $BOPC$  are cyclic. And since  $OP = OB$ , it follows that  $O$  is on the bisector of  $\angle PCB$ , as needed.

**Remark.** The angle equality is only used inasmuch  $\angle BAP$  is the average of  $\angle CBP$  and  $\angle BPC$ , i.e. only  $\frac{1+3}{2} = 2$  matters.