# IMO 1988/3 

## Evan Chen

## Twitch Solves ISL

Episode 24

## Problem

A function $f: \mathbb{N} \rightarrow \mathbb{N}$ is defined by

$$
\begin{aligned}
f(1) & =1, \quad f(3)=3 \\
f(2 n) & =f(n) \\
f(4 n+1) & =2 f(2 n+1)-f(n) \\
f(4 n+3) & =3 f(2 n+1)-2 f(n)
\end{aligned}
$$

for all positive integers $n$. Determine with proof the number of positive integers $n \leq 1988$ for which $f(n)=n$.

## Video

https://youtu.be/S6igQE958jQ

## External Link

https://aops.com/community/p365112

## Solution

The main claim is following.
Claim. $f(n)$ is equal to the result when $n$ is written in binary and its digits are reversed. Proof. Follows directly by induction.

So the question asks for the number of binary palindromes which are at most $1988=$ $11111000100_{2}$.

For $k=1,2, \ldots, 10$ there are $2^{[k / 2\rceil-1}$ binary palindromes with $k$ bits (note the first bit must be 1). For $k=11$, the number of binary palindromes which are also less than 1988 is $2^{5}-2$ (only 11111011111 and 11111111111 are missing).

Hence the final count is

$$
2^{0}+2^{0}+2^{1}+2^{1}+2^{2}+2^{2}+2^{3}+2^{3}+2^{4}+2^{4}+\left(2^{5}-2\right)=92 .
$$

