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Twitch Solves ISL

Episode 20

Problem

Let n be a positive integer and a_1, a_2, ..., a_{2n} are $2n$ distinct integers. Given the equation

$$|x - a_1|\,|x - a_2|\ldots|x - a_{2n}| = (n!)^2$$

has an integer solution, determine its value in terms of the a_i.

Solution

The left-hand side has $2n$ different nonzero numbers. So the absolute values are at least $n!^2$; hence it follows that the a_i are a shift of $(-n, -(n-1), \ldots, -1, 1, \ldots, n)$.

One of several valid functions is to take x to be the average of the a_i.