Singapore 2017/J2/2 Evan Chen

TWITCH SOLVES ISL

Episode 20

Problem

Let n be a positive integer and a_1, a_2, \ldots, a_{2n} are 2n distinct integers. Given the equation

 $|x - a_1| |x - a_2| \dots |x - a_{2n}| = (n!)^2$

has an integer solution, determine its value in terms of the a_i .

Solution

The left-hand side has 2n different nonzero numbers. So the absolute values are at least $n!^2$; hence it follows that the a_i are a shift of $(-n, -(n-1), \ldots, -1, 1, \ldots, n)$.

One of several valid functions is to take x to be the average of the a_i .