# KJMO 2012/1 

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## Twitch Solves ISL

Episode 19

## Problem

Let $a, b, c$ be positive real numbers satisfying $a b+b c+c a=1$. Prove that

$$
\frac{a+b}{\sqrt{a b(1-a b)}}+\frac{b+c}{\sqrt{b c(1-b c)}}+\frac{c+a}{\sqrt{c a(1-c a)}} \leq \frac{\sqrt{2}}{a b c}
$$

## Video

https://youtu.be/lC1g-01Me-w

## Solution

Since $1-a b=c(a+b)$ the inequality rewrites as

$$
\sum_{\mathrm{cyc}} \sqrt{a+b} \leq \sqrt{\frac{2}{a b c}}(a b+b c+c a)
$$

or simply

$$
\sum_{\mathrm{cyc}} \sqrt{\frac{a+b}{2}} \leq \sum_{\mathrm{cyc}} \sqrt{\frac{a b}{c}}
$$

If we let $x=\sqrt{\frac{b c}{a}}$, etc., then the inequality rewrites as

$$
\sum_{\mathrm{cyc}} \sqrt{\frac{x(y+z)}{2}} \leq x+y+z
$$

This follows by using AM-GM

$$
\sqrt{\frac{x(y+z)}{2}} \leq \frac{1}{2}\left[x+\frac{y+z}{2}\right]
$$

and summing cyclically.

