# Iran TST 2018/9 Evan Chen

TWITCH SOLVES ISL

Episode 14

## Problem

Let  $a_1, a_2, a_3, \ldots$  be an infinite sequence of distinct integers. Prove that there are infinitely many primes p that distinct positive integers i, j, k can be found such that  $p \mid a_i a_j a_k - 1$ .

### Video

https://youtu.be/\_o8r5wGUmWE

## **External Link**

https://aops.com/community/p10206683

#### Solution

We proceed by contradiction. Say a set S of integers is *prime-deficient* if at most finitely many primes divide one of its element. Then:

- The problem says  $\{a_1a_2a_k 1\}_k$  prime deficient.
- Hence  $\{a_1a_2a_3a_k a_3\}$  is prime deficient.
- By Kobayashi theorem, by adding  $a_3 a_2$ , we find  $\{a_1a_2a_3a_k a_2\}$  is not prime deficient.
- Hence  $\{a_1a_3a_k 1\}$  is not prime deficient.

This gives a contradiction.