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Twitch Solves ISL

Episode 14

Problem

Let $a_1, a_2, a_3, \cdots$ be an infinite sequence of distinct integers. Prove that there are infinitely many primes p that distinct positive integers i, j, k can be found such that $p \mid a_i a_j a_k - 1$.

Video

https://youtu.be/_o8r5wGUmWE

Solution

We proceed by contradiction. Say a set S of integers is *prime-deficient* if at most finitely many primes divide one of its element. Then:

- The problem says $\{a_1a_2a_k - 1\}_k$ prime deficient.
- Hence $\{a_1a_2a_3a_k - a_3\}$ is prime deficient.
- By Kobayashi theorem, by adding $a_3 - a_2$, we find $\{a_1a_2a_3a_k - a_2\}$ is not prime deficient.
- Hence $\{a_1a_3a_k - 1\}$ is not prime deficient.

This gives a contradiction.