# Iran TST 2018/9 

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## Twitch Solves ISL

Episode 14

## Problem

Let $a_{1}, a_{2}, a_{3}, \ldots$ be an infinite sequence of distinct integers. Prove that there are infinitely many primes $p$ that distinct positive integers $i, j, k$ can be found such that $p \mid a_{i} a_{j} a_{k}-1$.

## Video

https://youtu.be/_o8r5wGUmWE

## External Link

https://aops.com/community/p10206683

## Solution

We proceed by contradiction. Say a set $S$ of integers is prime-deficient if at most finitely many primes divide one of its element. Then:

- The problem says $\left\{a_{1} a_{2} a_{k}-1\right\}_{k}$ prime deficient.
- Hence $\left\{a_{1} a_{2} a_{3} a_{k}-a_{3}\right\}$ is prime deficient.
- By Kobayashi theorem, by adding $a_{3}-a_{2}$, we find $\left\{a_{1} a_{2} a_{3} a_{k}-a_{2}\right\}$ is not prime deficient.
- Hence $\left\{a_{1} a_{3} a_{k}-1\right\}$ is not prime deficient.

This gives a contradiction.

