

Twitch Ep6

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TWITCH SOLVES ISL

Episode 6

Problem

Let ABC be a triangle and let T be the contact point of the A -mixtilinear incircle with the circumcircle, and let T' be the reflection of T over BC . Prove that the nine-point circle of $T'BC$ is tangent to the incircle.

- Next, we note that:

$$\begin{aligned}
 \angle BMI &= \angle BQA \\
 &= \angle QAC + \angle QCA \\
 &= \angle KCB + \angle BCA \\
 &= \angle K'CB + \angle BCA \\
 &= \angle TCA \\
 &\implies \angle BMI = \angle TCA.
 \end{aligned}$$

- As $\angle KMK' = 2\angle KCT$ and $\angle KMY = 2\angle KCY$ we get

$$\implies \frac{1}{2}\angle K'MY = \angle TCA \implies \angle BMI = \frac{1}{2}\angle K'MY.$$

- Now, since $\angle KMB = \angle K'MB$, using the above result we get $\angle YMI = \angle KMI \implies \overline{MI}$ is the bisector of $\angle YMK$. e also had $MK = MY$, so $IK = IY$.

□

And thus, the nine-point circle of $\triangle T'BC$ is tangent to the incircle of $\triangle ABC$ (by appealing to Feuerbach theorem).