# Shortlist 2007 G8 

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## Twitch Solves ISL

Episode 4

## Problem

Point $P$ lies on side $A B$ of a convex quadrilateral $A B C D$. Let $\omega$ be the incircle of triangle $C P D$. Suppose that $\omega$ is tangent to the incircles of $\triangle A D P$ and $\triangle B P C$ at points $K$ and $L$, respectively. Let $E=\overline{A C} \cap \overline{B D}$ and $F=\overline{A K} \cap \overline{B L}$. Prove that line $E F$ passes through the center of $\omega$.

## Video

https://youtu.be/syyRZfWhoyI

## External Link

https://aops.com/community/p1186805

## Solution

We begin with the following claim.
Claim. The quadrilaterals $A P C D$ and $P B C D$ both have an incircle.
Proof. Follows by Pitot theorem since $A P-A D=K P-K D=C P-C D$ (see SL 2017 G7 for another example of this lemma).

Let $I$ be the center of $\omega$ (the incenter of $\triangle C P D$ ). Moreover, if we let $G=\overline{A D} \cap \overline{B C}$, we will denote by $J$ the incenter of $\triangle G B A$, and by $(J)$ the circumcircle.


Claim. The point $F$ is the insimilicenter of $(I)$ and $(J)$.
Proof. By Monge d'Alembert theorem on $(I)$, $(J)$, and the incircle of $\triangle A P B$, we get that $\overline{B L}$ passes through said insimilicenter. By symmetry, so does $\overline{C K}$, as needed.

Claim. The point $E$ is the exsimilicenter of $(I)$ and $(J)$.
Proof. By Monge d'Alembert theorem on (I), (J), and the incircle of BPDC, we get that $\overline{B D}$ passes through said exsimilicenter. By symmetry, so does $\overline{C A}$, as needed.

The previous two claims imply all four points $E, F, I, J$ are collinear.

