

# Shortlist 2007 G2

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TWITCH SOLVES ISL

Episode 2

## Problem

Denote by  $M$  midpoint of side  $BC$  in an isosceles triangle  $\triangle ABC$  with  $AC = AB$ . Take a point  $X$  on a smaller arc  $\widehat{MA}$  of circumcircle of triangle  $\triangle ABM$ . Denote by  $T$  point inside of angle  $BMA$  such that  $\angle TMX = 90^\circ$  and  $TX = BX$ .

Prove that  $\angle MTB - \angle CTM$  does not depend on choice of  $X$ .

## Video

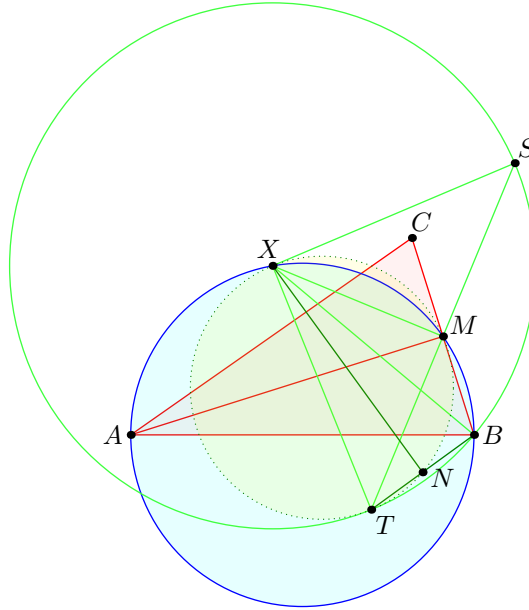
<https://youtu.be/vA1iwW41Jmw>

## External Link

<https://aops.com/community/p1143932>

## Solution

Construct parallelogram  $CTBS$  whose diagonals meet at  $M$ . Also, let  $N$  be the midpoint of  $\overline{BT}$ .



We first eliminate  $C$  from the diagram by noting that

$$\angle CTM = \angle MSB = \angle TSB = \frac{1}{2}\angle TXB = \angle NXB.$$

Also, noting that  $XMTN$  are cyclic (as  $\angle XMT = \angle XNT = 90^\circ$ ), we have

$$\angle MTB = \angle MTN = \angle NXM.$$

Thus  $\angle MTB - \angle CTM = \angle NXM - \angle NXB = \angle MXB$  which is fixed.