Shortlist 2007 C1

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Twitch Solves ISL

Episode 2

Problem

Let $n \geq 1$ be an integer. Find all sequences $a_1, a_2, \ldots, a_{n^2+n}$ consisting of 0 and 1 such that

$$a_{i+1} + a_{i+2} + \cdots + a_{i+n} < a_{i+n+1} + a_{i+n+2} + \cdots + a_{i+2n}$$

for all $0 \leq i \leq n^2 - n$.

External Link

https://aops.com/community/p1187174

Solution

We give an example for $n = 5$ which generalizes readily:

$$00000 \mid 00001 \mid 00011 \mid 00111 \mid 01111 \mid 11111.$$

It's obvious this works. One can actually prove this is the only one.

Now:

- First, split the $n^2 + n$ numbers into $n + 1$ blocks of size n (as in the example above). Then evidently, they must have 0, 1, ..., n ones in that order.
- TODO finish this up.