Mock IMO Day 1

MOP 2025

June 14, 2025

Time limit: 4.5 hours. We hope you're looking forward to your first free day tomorrow!

1. Let n be a positive integer. Find the minimum possible value of

$$S = 2^0 x_0^2 + 2^1 x_1^2 + \cdots + 2^n x_n^2,$$

   where x_0, ..., x_n are nonnegative integers such that $x_0 + \cdots + x_n = n$.

2. Let $ABCD$ be a quadrilateral with AB parallel to CD and $AB < CD$. Lines AD and BC intersect at a point P. Point X distinct from C lies on the circumcircle of triangle ABC such that $PC = PX$. Point Y distinct from D lies on the circumcircle of triangle ABD such that $PD = PY$. Lines AX and BY intersect at Q.

   Prove that PQ is parallel to AB.

3. Let n be a positive integer. We say that a polynomial P with integer coefficients is n-*good* if there exists a polynomial Q of degree 2 with integer coefficients such that $Q(k)(P(k) + Q(k))$ is never divisible by n for any integer k.

   Determine all integers n such that every polynomial with integer coefficients is n-good.

Mock IMO Day 2

MOP 2025

June 21, 2025

Time limit: 4.5 hours. Did you know MOP is half over?

4. Determine all finite, nonempty sets $\mathcal{S}$ of positive integers such that for every a, $b \in \mathcal{S}$ there exists $c \in \mathcal{S}$ with $a \mid b + 2c$.

5. Let N be a positive integer. Geoff and Ceri play a game in which they start by writing the numbers 1, 2, …, N on a board. They then take turns to make a move, starting with Geoff. Each move consists of choosing a pair of integers (k, n), where $k \geq 0$ and n is one of the integers on the board, and then erasing every integer s on the board such that $2^k \mid n - s$. The game continues until the board is empty. The player who erases the last integer on the board loses.

   Determine all values of N for which Geoff can ensure that he wins, no matter how Ceri plays.

6. Let $a_0 < a_1 < a_2 < \cdots$ be an infinite strictly increasing sequence of positive integers such that for each $n \geq 1$, a_n is either $\frac{a_{n-1}+a_{n+1}}{2}$ or $\sqrt{a_{n-1}a_{n+1}}$.

   Define $b_n = A$ if $a_n = \frac{a_{n-1}+a_{n+1}}{2}$ and $b_n = G$ otherwise. Prove that b_n is eventually periodic.