Mock IMO

Day: **1**

Monday, July 20, 2020

Mock IMO in July feels a bit too real?

Problem 1. You're given n blocks each with weight at least 1 and total weight $2n$. Prove that for every real number $r \in [0, 2n-2]$ there is a subset of the blocks whose total weight is between r and $r+2$ inclusive.

Problem 2. Let S be a set of integers for which $2^a - 2^b \in S$ for any positive integers a and b. Suppose that for any $a_0, \ldots, a_n \in S$, all integer roots of the polynomial $a_0 + a_1 x + \cdots + a_n x^n$ are also in S. Show that $S = \mathbb{Z}$.

Problem 3. Let $\mathcal{L}$ be the set of all lines in the plane and let f be a function that assigns to each line $\ell \in \mathcal{L}$ a point $f(\ell)$ on ℓ. Suppose that for any point X and for any three lines ℓ_1, ℓ_2, ℓ_3 passing through X, the points $f(\ell_1)$, $f(\ell_2)$, $f(\ell_3)$, and X lie on a circle. Prove that there is a unique point P such that $f(\ell) = P$ for any line ℓ passing through P.

Time limit: 4 hours and 30 minutes.
Each problem is worth seven points.

Mock IMO

Day: **2**

Tuesday, July 21, 2020

Sorry, no medium geometry.

Problem 4. Let P be a point inside a triangle ABC. Let $A_1 = \overline{AP} \cap \overline{BC}$, and let A_2 denote the reflection of P over A_1. Define B_1, B_2, C_1, C_2 similarly. Prove that A_2, B_2, C_2 cannot all lie strictly inside the circumcircle of $\triangle ABC$.

Problem 5. Let $n \geq 2$ be an integer. Given real numbers $a_1, \ldots, a_n$ with sum zero, prove that

$$\sum_{\substack{1 \leq i < j \leq n \\ |a_i - a_j| \geq 1}} a_i a_j < 0$$

whenever the sum on the left-hand side is nonempty.

Problem 6. Prove that if a and b are positive integers, then $a^2 + \left\lceil \frac{4a^2}{b} \right\rceil$ isn't a square.

Time limit: 4 hours and 30 minutes.
Each problem is worth seven points.