Mock IMO

Day: **1**

Saturday, June 8, 2019

Problem 1. Determine all pairs (m, n) of positive integers for which there exists a positive integer s such that sm and sn have an equal number of divisors.

Problem 2. A point T is chosen inside a triangle ABC. Let A_1, B_1, C_1 be the reflections of T in BC, CA, and AB, respectively. Let Ω be the circumcircle of $\triangle A_1B_1C_1$. The lines A_1T, B_1T, C_1T meet Ω again at A_2, B_2, C_2, respectively. Prove that lines AA_2, BB_2, CC_2 meet on Ω.

Problem 3. Let k be a positive integer. Evan is organizing a round-robin StarCraft tournament with $2k$ players. The tournament is played over $\binom{2k}{2}$ consecutive days, with one match on each day. Each player arrives at the hotel on the day of their first match, and leaves on the day of their last match. For each day, Evan must pay 1 coin per player in the hotel (including arrival and departure days). Determine the least possible number of coins Evan must pay.

Time limit: 4 hours and 30 minutes.
Each problem is worth seven points.

Mock IMO

Day: 2

Saturday, June 16, 2019

(From the other side of the testing room)
Yang: This is trash! #4 inequality, asymmetric, nonhomogeneous, with a 7 and a radical!
(Yang flips his chair and takes off his shirt)
I'm walking out!

Mock IMO Day 2 from MOP 2015 closing skit

Problem 4. A circle ω of radius 1 is given. A collection T of triangles is *good* if

- each triangle in T is inscribed in ω;
- no two triangles in T share a common interior point.

Determine all $t > 0$ such that, for any n, there exists a good collection of n triangles, all with perimeter greater than t.

Problem 5. Let $n \geq 2$ be a given positive integer. Steve performs a sequence of turns on a board consisting of $n+1$ squares in a row, numbered 0 to n from left to right. Initially, n stones are put into square 0, and the other squares are empty. At every turn, Steve chooses any nonempty square, say with k stones, takes one of the stones, and moves it to the right by at most k squares (the stones should stay within the board). Steve's aim is to move all n stones to square n.

Let a_n denote the minimum number of turns required for Steve to reach his goal. Prove that $a_n = \Theta(n \log n)$.

Problem 6. What is the maximum possible value of

$$S = \sqrt[3]{\frac{a}{b+7}} + \sqrt[3]{\frac{b}{c+7}} + \sqrt[3]{\frac{c}{d+7}} + \sqrt[3]{\frac{d}{a+7}}$$

over nonnegative real numbers a, b, c, d with sum 100?

Time limit: 4 hours and 30 minutes.
Each problem is worth seven points.