# Mock IMO

Day: **1**

*Saturday, June 10, 2017*

Not a real IMO.

**Problem 1.** Let $ABC$ be a triangle with $AB = AC \neq BC$ and let $I$ denote the incenter. The line $BI$ meets $AC$ at $D$, and the line through $D$ perpendicular to $AC$ meets $AI$ at $E$. Prove that the reflection of $I$ in $\overline{AC}$ lies on the circumcircle of triangle $BDE$.

**Problem 2.** Let $n \geq 3$ be a positive integer. Determine the maximal number of diagonals of a regular $n$-gon that can be drawn such that: any two drawn diagonals which intersect in the interior of the $n$-gon are perpendicular.

**Problem 3.** Find all polynomials $P(x)$ of odd degree $d$ with integer coefficients such that for any positive integer $N$, there exist $N$ distinct positive integers $x_1, x_2, \ldots, x_N$ with the following properties.

- $\frac{1}{2} < \frac{P(x_i)}{P(x_j)} < 2$ for all $1 \leq i, j \leq N$, and
- $\frac{P(x_i)}{P(x_j)}$ is the $d$th power of a rational number for every $1 \leq i, j \leq N$.

*Time limit: 4 hours and 30 minutes.*
*Each problem is worth seven points.*

*Saturday, June 17, 2017*

Geoff is back!

**Problem 4.** Let $\tau(n)$ be the number of positive divisors of $n$. Let $\tau_1(n)$ be the number of positive divisors of $n$ which have remainders 1 when divided by 3. Find all positive integral values of the fraction $\frac{\tau(10n)}{\tau_1(10n)}$.

**Problem 5.** Find all functions $f : (0, \infty) \to (0, \infty)$ such that for any $x, y \in (0, \infty)$,

$$xf(x^2)f(f(y)) + f(yf(x)) = f(xy)\left(f(f(x^2)) + f(f(y^2))\right).$$

**Problem 6.** Geoff has a connected simple graph $G$ on $n \geq 3$ vertices. Every year, Geoff deletes an edge $vw$ from $G$. At the same time, for any other vertex $x \neq v, w$ adjacent to exactly one of $v$ and $w$, Geoff adds an edge between $x$ and the other vertex. Suppose that for any partition of the $n$ vertices into two nonempty groups, Geoff deletes an edge between these two groups infinitely many times (not necessarily the same edge). Prove that eventually, at some point in time, there will be a vertex connected by an edge to all other vertices.

*Time limit: 4 hours and 30 minutes.*
*Each problem is worth seven points.*