Conundrum OPAL Hunt 2

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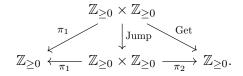
Conundrum

Normally, puzzle hunts don't give instructions. This conundrum is much easier! It consists of nothing but instructions!

- 1. Write down five blanks on a sheet of paper.
- 2. For a function f, let $f^{-1}(y)$ denote the unique x in its domain such that f(x) = y (this notation will only be used when there is exactly one such x).
- 3. For any integer n define the shorthand

$$f^{n} = \begin{cases} f \text{ applied } n \text{ times} & n > 0\\ \text{id} & n = 0\\ f^{-1} \text{ applied } |n| \text{ times} & n < 0. \end{cases}$$

- 4. Let $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ denote the set of ordered pairs of nonnegative integers.
- 5. Define sequence 79, 0-indexed, as the sequence of powers of 2.
- 6. Define a function Get: $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0}$ by mapping (a, b) to the bth element of sequence a. (This is undefined if b = 0 and sequence a is 1-indexed.)
- 7. Check your understanding by verifying that Get(79, 10) = 1024.
- 8. Fill the first blank with the value of Get(79, 4) Get(79, 0).
- 9. For i = 1, 2, let $\pi_i : \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0}$ denote the projection onto the *i*th component. That is, $\pi_1(a, b) = a$ and $\pi_2(a, b) = b$.
- 10. Define sequence 142, 0-indexed, such that Get(142, n) is the number of bijective functions from $\{1, \ldots, n\}$ to itself for all n.
- 11. Define Jump: $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ such that there's a commutative diagram



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12. Check your understanding by verifying that

$$\pi_2(\text{Jump}^2(142,4)) = 620448401733239439360000.$$

13. Fill the third blank with the value of

$$\sum_{\varnothing \neq T \subseteq S} \operatorname{Get} \left(142, \sum_{t \in T} t \right)$$

where S denotes the set of $z \in \mathbb{Z}_{\geq 0}$ for which (142, z) is a fixed point of Jump.

- 14. We say a pair (a, b) is *good* if b only appears once in sequence a. When that occurs, let Rev(a, b) be the unique c such that Get(a, c) = b.
- 15. Define sequence 40, 1-indexed, by 2, 3, 5, 7, 11, ..., and so on.
- 16. Define a function Next: $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ for which $\pi_1 \circ \text{Next} = \pi_1$, and if (a,b) is good then $\pi_2(\text{Next}(a,b)) = \text{Get}(a,\text{Rev}(a,b)+1)$.
- 17. Check your understanding by verifying that $Next^{25}(40,2) = (40,101)$.
- 18. Fill the fourth blank with the value of $\pi_2(\text{Next}^{-3}(40,31))$.
- 19. Define sequence 108, 0-indexed, such that Get(108,0) = Get(108,1) = 1 and

$$Get(108, n) = \sum_{i=0}^{n-1} Get(108, i) Get(108, n - 1 - i).$$

- 20. Fill the second blank with the value of $\pi_2(\text{Jump}^{-2}(108, 39044429911904443959240))$.
- 21. Replace each of the four filled blanks with corresponding letters via A1Z26.
- 22. Check your understanding by verifying that Get(290, 45) = Get(292, 22) + 1.
- 23. Define a function Down: $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ such that for all good (a, b),

$$Down(a, b) = (a + 1, Get(a + 1, Rev(a, b))).$$

- 24. Check your understanding by verifying that $Down^2(40, 29) = (42, 11111111111)$.
- 25. Define a function Seek: $\mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ such that $\pi_2 \circ \operatorname{Seek} = \pi_2$ and $\pi_1(\operatorname{Seek}(a,b)) = \max \{c < a \mid \exists d : \operatorname{Get}(c,d) = b\}$.
- 26. Check your understanding by verifying that Seek(43,61) = (40,61).
- 27. Fill the fifth blank with

$$\pi_2(\operatorname{Next}^{-1}(50128,\pi_2(\operatorname{Next}(\operatorname{Seek}(\operatorname{Next}^{-2}(\operatorname{Jump}(\operatorname{Down}^{-2}(\operatorname{Get}(10,293),10)))))))).$$

28. The blanks should have four letters followed by a number. Call this in!